Inequalities and modulus graphs

Skills sheet · HL AA · AHL 2.15, 2.16

KEY FACTS

One side zero, factorise, critical values, signs P1
$|f(x)|$: reflect below in the x-axis D1
$f(|x|)$: reflect $x \geq 0$ in the y-axis D1
$|f| = g$: $f = g$ or $f = -g$, then check $g \geq 0$ P2

Never multiply by a sign you do not know W1.
$|x - a| < b$: *one interval* W2.
$|x - a| > b$: *two intervals* W2.

WHAT UNLOCKS

- a fraction in an inequality $\rightarrow$ subtract, one fraction, then the sign table.
- a cubic inequality $\rightarrow$ factorise fully, sketch the cubic.
- "how many solutions of $|f(x)| = k$" $\rightarrow$ the graph of $|f|$ and a horizontal line.
- $|f(x)| \geq g(x)$ with no easy algebra $\rightarrow$ the GDC: intersections, then intervals.

WORKED EXAMPLE ①

Solve $\dfrac{x-1}{x+2} > 0$. no GDC

critical values 1 and -2; test $x = 0$: $-\frac{1}{2} < 0$ (M1)
$x < -2$ or $x > 1$ A1

Method
A sign table
① Zeros above and below.
② One test value per interval.
③ Denominator zero: excluded.

WORKED EXAMPLE ②

Solve $|2x - 1| = x + 4$. no GDC

$2x - 1 = x + 4$: $x = 5$; $2x - 1 = -(x + 4)$: $x = -1$ M1A1
check: $x + 4 = 9$ and 3, both ≥ 0: $x = 5$, $x = -1$ R1

Method
A modulus equation
① Two cases, $+g$ and $-g$.
② Solve each.
③ Reject any with $g < 0$.

FLUENCY

1. $|x - 2| < 5$ no GDC

2. $|x + 1| \geq 2$ no GDC

3. $x^2 < 9$ no GDC

4. $(x - 3)(x + 1) \leq 0$ no GDC

5. The critical values of $\dfrac{x+5}{x-2} \geq 0$ no GDC

6. $f(|x|)$ at $x = -2$, for $f(x) = x - 3$ no GDC

REASONING

●●● **Question 1.** Answer without a calculator. [no GDC]

(a) Solve the inequality $\dfrac{1}{x-1} < 2$. [4]

●●● **Question 2.** Let $p(x) = x^3 - 5x^2 + 2x + 8$. [no GDC]

(a) Show that $x + 1$ is a factor of $p(x)$, and hence solve $p(x) \leq 0$. [4]

●●● **Question 3.** Answer without a calculator. [no GDC]

(a) Solve the inequality $|3x - 2| < x + 2$. [4]

PROBLEMS

●●● **Question 4.** Let $f(x) = x^2 - 6x + 5$. [no GDC]

(a) Find the x-intercepts of the graph of f and the coordinates of its vertex. [2]

(b) Find the values of k for which the equation $|f(x)| = k$ has exactly four solutions, and the value of k for which it has exactly three. [4]

●●● **Question 5.** Let $f(x) = x^2 - 4x$. [no GDC]

(a) Find the x-intercepts and the minimum points of the graph of $y = f(|x|)$. [3]

(b) Solve the equation $f(|x|) = -3$. [3]

ANSWERS

1 $-3 < x < 7$

2 $x \leq -3$ or $x \geq 1$

3 $-3 < x < 3$

4 $-1 \leq x \leq 3$

5 -5 and 2

6 $f(2) = -1$

Q1 $\frac{1}{x-1} - 2 = \frac{3-2x}{x-1} < 0$: $x < 1$ or $x > \frac{3}{2}$

Q2 $p(-1) = 0$; $(x+1)(x-2)(x-4) \leq 0$: $x \leq -1$ or $2 \leq x \leq 4$

Q3 $-(x+2) < 3x - 2 < x + 2$: $0 < x < 2$

Q4 $1, 5$; $(3, -4)$; the maximum of $|f|$ between the roots is 4: four solutions for $0 < k < 4$, three for $k = 4$

Q5 $x^2 - 4|x|$: intercepts $-4, 0, 4$; minima $(\pm 2, -4)$; $|x| = 1$ or $|x| = 3$: $x = \pm 1, \pm 3$

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