Marked example 3

Paper 2 style · Graphic display calculator

3 6 marks AHL 2.14 ●●○ Typical

Let $f(x) = x^2 - 4x + 1$, for $x \geq k$.

(a) Find the smallest value of k for which f has an inverse. **[1]**

(b) For this value of k, find $f^{-1}(x)$ and state its domain. **[3]**

(c) Solve the equation $f(x) = f^{-1}(x)$. **[2]**

MARKSCHEME (a)

$k = 2$ **A1**

Total 1 mark

Note The vertex.

MARKSCHEME (b)

$y = (x-2)^2 - 3,\ x = 2 + \sqrt{y+3}$ **M1**

$f^{-1}(x) = 2 + \sqrt{x+3}$ **A1**

$x \geq -3$ **A1**

Total 3 marks

Note Completing the square, or the quadratic formula; the $+$ root because $x \geq 2$.

MARKSCHEME (c)

$x^2 - 4x + 1 = x$ **(M1)**

$x = 4.79$ **A1**

Total 2 marks

Note f is increasing, so f and f^{-1} meet on $y = x$.

Note Exact value $\frac{5+\sqrt{21}}{2}$; the other root, 0.209, is not in the domain.

Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software.
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