Odd, even and self-inverse functions

Skills sheet · HL AA · AHL 2.14

KEY FACTS

Even: $f(-x) = f(x)$; odd: $f(-x) = -f(x)$ D1
Like signs: even × even, odd × odd are even P1
Not one-to-one: restrict from the vertex P2
Self-inverse: $f(f(x)) = x$, symmetric in $y = x$ D2

A general x, not one value W1.
$f^{-1}(x) \neq \frac{1}{f(x)}$ W2.
f increasing: meets f^{-1} on $y = x$ W2.

WHAT UNLOCKS

- "odd, even or neither" → $f(-x)$ simplified; neither: one counterexample.
- $\int_{-a}^{a}$ of an odd function → 0, by symmetry.
- "the inverse" of a quadratic → complete the square, keep the root that fits the domain.
- $\frac{ax+b}{cx-a}$ → self-inverse: check $f(f(x)) = x$.

WORKED EXAMPLE ①

Is $f(x) = x \sin x$ odd, even or neither? no GDC

$f(-x) = (-x)\sin(-x) = (-x)(-\sin x)$ M1
$= x \sin x = f(x)$: even A1

Method
Odd or even
① Replace x by $-x$ everywhere.
② $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$.
③ Compare with $f(x)$.

WORKED EXAMPLE ②

Find the inverse of $f(x) = (x-1)^2 + 2$, $x \geq 1$. no GDC

$x = 1 + \sqrt{y-2}$ (the + root, since $x \geq 1$) M1
$f^{-1}(x) = 1 + \sqrt{x-2}$, $x \geq 2$ A1A1

Method
A restricted inverse
① Make x the subject.
② Choose the sign from the domain.
③ Domain of f^{-1}: range of f.

FLUENCY

●○○ **1.** $x^4 - x^2$: odd, even or neither? no GDC

●○○ **2.** $x^3 \cos x$: odd, even or neither? no GDC

●○○ **3.** $x + x^2$: odd, even or neither? no GDC

●○○ **4.** $\frac{\sin x}{x}$: odd, even or neither? no GDC

●○○ **5.** Is $f(x) = 5 - x$ self-inverse? no GDC

●○○ **6.** $\frac{1}{x^3}$: odd, even or neither? no GDC

REASONING

●●● **Question 1.** Let $f(x) = \dfrac{x}{x^2+4}$, for $x \in \mathbb{R}$. [no GDC]

(a) Show that f is odd, and hence write down the value of $\displaystyle\int_{-3}^{3} f(x)\,\mathrm{d}x$. [3]

●●● **Question 2.** Let $f(x) = x^2 - 6x + 10$, for $x \geq 3$. [no GDC]

(a) Find $f^{-1}(x)$ and state its domain. [4]

●●● **Question 3.** Let $f(x) = \dfrac{4x-1}{x-4}$, for $x \neq 4$. [no GDC]

(a) Show that f is self-inverse. [3]

PROBLEMS

●●● **Question 4.** Let $f(x) = \dfrac{ax+3}{2x-5}$, for $x \neq \frac{5}{2}$, where $a \in \mathbb{R}$. [no GDC]

(a) Find the value of a for which f is self-inverse. [3]

(b) For this value of a, find the x-coordinates of the points where the graph of f meets the line $y = x$. [2]

●●● **Question 5.** Let $f(x) = x^2 - 2x$, for $x \geq 1$, and let $g = f^{-1}$. [no GDC]

(a) Find $g(x)$ and state its domain. [3]

(b) Solve the equation $f(x) = g(x)$. [2]

ANSWERS

1 even

2 odd (odd $\times$ even)

3 neither: $f(-1) = 0$, $f(1) = 2$

4 even (odd $\div$ odd)

5 yes: $5 - (5 - x) = x$

6 odd

Q1 $f(-x) = \frac{-x}{x^2+4} = -f(x)$; the integral is 0

Q2 $(x-3)^2 + 1$: $f^{-1}(x) = 3 + \sqrt{x-1}$, $x \geq 1$

Q3 $f(f(x)) = \frac{4(4x-1)-(x-4)}{(4x-1)-4(x-4)} = \frac{15x}{15} = x$

Q4 $a = 5$ ($f(f(x)) = x$ needs $a = -(-5)$); $5x + 3 = 2x^2 - 5x$: $x = \frac{5 \pm \sqrt{31}}{2}$

Q5 $(x-1)^2 - 1$: $g(x) = 1 + \sqrt{x+1}$, $x \geq -1$; f increasing, so $x^2 - 2x = x$: $x = 3$ ($x = 0$ is not in the domain)

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