Partial fractions

Skills sheet · HL AA · AHL 1.11

KEY FACTS

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$ D1

Multiply out, then $x = -\frac{b}{a}$, $x = -\frac{d}{c}$ P1

$\int \frac{A}{ax+b}\,\mathrm{d}x = \frac{A}{a}\ln|ax+b| + c$ P2

An unknown limit: one logarithm, then solve P3

Check with $x = 0$ W1.

Divide by the coefficient of x W2.

$\int \frac{1}{3-x} = -\ln|3-x|$ W2.

WHAT UNLOCKS

- "express in partial fractions" → the identity $px+q = A(\ldots) + B(\ldots)$, then the two roots.
- "hence find the integral" → a logarithm for each fraction.
- "in the form $\ln k$" → $p\ln u + q\ln v = \ln u^p v^q$.
- the value of the integral given → an equation in the limit k.

WORKED EXAMPLE ①

Express $\frac{5x+1}{(x-1)(x+2)}$ in partial fractions. no GDC

$5x+1 = A(x+2) + B(x-1)$ M1

$x=1$: $A=2$; $x=-2$: $B=3$: $\frac{2}{x-1} + \frac{3}{x+2}$ A1A1

Method

Partial fractions

① One fraction per factor.
② Multiply by the denominator.
③ Substitute each root.

WORKED EXAMPLE ②

Find $\int \left(\frac{2}{x-1} + \frac{3}{2x+1}\right)\mathrm{d}x$. no GDC

$2\ln|x-1| + \frac{3}{2}\ln|2x+1| + c$ A1A1

Method

Integrating

① Each fraction alone.
② $\frac{A}{a}$ in front of the ln.
③ One constant c.

FLUENCY

●○○ **1.** $\frac{1}{x-1} + \frac{2}{x+1}$ as one fraction no GDC

●○○ **2.** $\frac{4}{(x-1)(x+3)}$ in partial fractions no GDC

●○○ **3.** $\frac{x}{(x+2)(x+3)}$ in partial fractions no GDC

●○○ **4.** $\int \frac{6}{3x+2}\,\mathrm{d}x$ no GDC

●○○ **5.** $\int \frac{1}{5-x}\,\mathrm{d}x$ no GDC

●○○ **6.** $2\ln 3 - \ln 6$ as one logarithm no GDC

REASONING

●●● **Question 1.** Let $f(x) = \dfrac{3x+5}{(x+1)(x+3)}$. [no GDC]

(a) Express $f(x)$ in partial fractions. [3]

●●● **Question 2.** Let $g(x) = \dfrac{2x-3}{(2x+1)(x-1)}$, for $x > 1$. [no GDC]

(a) Express $g(x)$ in partial fractions, and hence find $\displaystyle\int g(x)\,\mathrm{d}x$. [5]

●●● **Question 3.** Let $h(x) = \dfrac{1}{x(x+1)}$, for $x > 0$. [no GDC]

(a) Show that $\displaystyle\int_1^3 h(x)\,\mathrm{d}x = \ln\frac{3}{2}$. [5]

PROBLEMS

●●● **Question 4.** Let $f(x) = \dfrac{2}{(1+x)(1-x)}$, for $-1 < x < 1$. [no GDC]

(a) Express $f(x)$ in partial fractions. [2]

(b) Find the value of k, where $0 < k < 1$, for which $\displaystyle\int_0^k f(x)\,\mathrm{d}x = \ln 3$. [5]

●●● **Question 5.** Let $f(x) = \dfrac{x+5}{(x+1)(x-3)}$, for $x > 3$. [no GDC]

(a) Express $f(x)$ in partial fractions. [3]

(b) Show that $\displaystyle\int_4^7 f(x)\,\mathrm{d}x = \ln 10$. [4]

ANSWERS

1 $\dfrac{3x-1}{(x-1)(x+1)}$

2 $\dfrac{1}{x-1}-\dfrac{1}{x+3}$

3 $-\dfrac{2}{x+2}+\dfrac{3}{x+3}$

4 $2\ln|3x+2|+c$

5 $-\ln|5-x|+c$

6 $\ln\frac{9}{6}=\ln\frac{3}{2}$

Q1 $\frac{1}{x+1}+\frac{2}{x+3}$

Q2 $\frac{8/3}{2x+1}-\frac{1/3}{x-1}$; $\frac{4}{3}\ln|2x+1|-\frac{1}{3}\ln|x-1|+c$

Q3 $\frac{1}{x}-\frac{1}{x+1}$; $[\ln x-\ln(x+1)]_1^3=\ln\frac{3}{4}-\ln\frac{1}{2}=\ln\frac{3}{2}$

Q4 $\frac{1}{1+x}+\frac{1}{1-x}$; $\ln\frac{1+k}{1-k}=\ln 3$: $k=\frac{1}{2}$

Q5 $-\frac{1}{x+1}+\frac{2}{x-3}$; $[2\ln(x-3)-\ln(x+1)]_4^7=2\ln 4-\ln 8+\ln 5=\ln\frac{16\times5}{8}=\ln 10$

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