Complex numbers in Cartesian form

Skills sheet · HL AA · AHL 1.12

KEY FACTS

$z = a + b\mathrm{i}$, $\mathrm{i}^2 = -1$, $z^* = a - b\mathrm{i}$ D1

$\dfrac{u}{v} = \dfrac{uv^*}{vv^*}$, $zz^* = a^2 + b^2$ P1

Equal: real parts **and** imaginary parts P2

$|z| = \sqrt{a^2 + b^2}$, $-\pi < \arg z \leq \pi$ D2

Real coefficients: roots α, α^* P4

$\mathrm{Im}(z)$ has no i W1.

A sketch before the argument D2.

Conjugate pairs need real coefficients W2.

WHAT UNLOCKS

- z and z^* in one equation $\rightarrow$ $z = a + b\mathrm{i}$, then equate real and imaginary parts.
- a quotient $\rightarrow$ multiply above and below by the conjugate of the denominator.
- "purely imaginary" $\rightarrow$ real part $= 0$; "real" $\rightarrow$ imaginary part $= 0$.
- one non-real root and real coefficients $\rightarrow$ its conjugate, then the quadratic factor.

WORKED EXAMPLE ①

Write $\dfrac{4 - 3\mathrm{i}}{2 + \mathrm{i}}$ in the form $a + b\mathrm{i}$. no GDC

$$\frac{(4 - 3\mathrm{i})(2 - \mathrm{i})}{2^2 + 1^2} = \frac{8 - 4\mathrm{i} - 6\mathrm{i} + 3\mathrm{i}^2}{5} \quad \text{(M1)}$$

$$= \frac{5 - 10\mathrm{i}}{5} = 1 - 2\mathrm{i} \quad \text{A1}$$

Method

A quotient

① Conjugate of the denominator.
② Below: $a^2 + b^2$, real.
③ Split into $a + b\mathrm{i}$.

WORKED EXAMPLE ②

Solve $z^2 + 6z + 34 = 0$. no GDC

$$z = \frac{-6 \pm \sqrt{36 - 136}}{2} = \frac{-6 \pm \sqrt{-100}}{2} \quad \text{(M1)}$$

$$z = -3 \pm 5\mathrm{i} \quad \text{A1}$$

Method

A negative discriminant

① The formula, as usual.
② $\sqrt{-100} = 10\mathrm{i}$.
③ Two conjugate roots.

FLUENCY

●○○ **1.** $(3 - \mathrm{i})(2 + 5\mathrm{i})$ no GDC

●○○ **2.** i^7 no GDC

●○○ **3.** $|5 - 12\mathrm{i}|$ no GDC

●○○ **4.** Re and Im of $(1 + \mathrm{i})^2$ no GDC

●○○ **5.** $\arg(-1 + \mathrm{i})$ no GDC

●○○ **6.** $\dfrac{1}{1 - \mathrm{i}}$ in the form $a + b\mathrm{i}$ no GDC

REASONING

●●● **Question 1.** The complex number z satisfies $3z - 2z^* = 1 + 10\mathrm{i}$. [no GDC]

(a) Find z, giving your answer in the form $a + b\mathrm{i}$. [3]

●●● **Question 2.** Let $z = \dfrac{a + \mathrm{i}}{1 - \mathrm{i}}$, where $a \in \mathbb{R}$. [no GDC]

(a) Find the value of a for which z is purely imaginary, and the corresponding value of z. [4]

●●● **Question 3.** The roots of $z^2 - 4z + 29 = 0$ are z_1 and z_2. [no GDC]

(a) Find z_1 and z_2, and hence find $|z_1 - z_2|$. [4]

PROBLEMS

●●● **Question 4.** Let $w = -15 + 8\mathrm{i}$. [no GDC]

(a) Find the two square roots of w, giving your answers in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$. [4]

(b) Hence solve the equation $z^2 + z + 4 - 2\mathrm{i} = 0$. [3]

●●● **Question 5.** Consider the equation $z^3 + pz^2 + qz + 20 = 0$, where $p, q \in \mathbb{R}$. One root is $-1 + 3\mathrm{i}$. [no GDC]

(a) Find the value of p, the value of q and the other two roots. [6]

ANSWERS

1 $11 + 13\mathrm{i}$

2 $-\mathrm{i}$

3 13

4 $(1+\mathrm{i})^2 = 2\mathrm{i}$: $\mathrm{Re} = 0$, $\mathrm{Im} = 2$

5 $\frac{3\pi}{4}$ (second quadrant)

6 $\frac{1}{2} + \frac{1}{2}\mathrm{i}$

Q1 $3a + 3b\mathrm{i} - 2a + 2b\mathrm{i} = a + 5b\mathrm{i}$: $a = 1$, $b = 2$, $z = 1 + 2\mathrm{i}$

Q2 $z = \frac{(a+\mathrm{i})(1+\mathrm{i})}{2} = \frac{a-1}{2} + \frac{a+1}{2}\mathrm{i}$: $a = 1$, $z = \mathrm{i}$

Q3 $2 \pm 5\mathrm{i}$; $|z_1 - z_2| = |10\mathrm{i}| = 10$

Q4 $a^2 - b^2 = -15$, $ab = 4$, $a^4 + 15a^2 - 16 = 0$, $a^2 = 1$: $\pm(1 + 4\mathrm{i})$; $z = \frac{-1 \pm (1+4\mathrm{i})}{2}$: $2\mathrm{i}$, $-1 - 2\mathrm{i}$

Q5 $-1 - 3\mathrm{i}$ is a root; $(z+1)^2 + 9 = z^2 + 2z + 10$; $\times(z+2)$: $z^3 + 4z^2 + 14z + 20$: $p = 4$, $q = 14$, roots $-1 \pm 3\mathrm{i}$, -2

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