Counting and the extended binomial theorem

Skills sheet · HL AA · AHL 1.10

KEY FACTS

And then: multiply; or: add D1

$${}^nP_r = \frac{n!}{(n-r)!},\ \binom{n}{r} = \frac{n!}{r!(n-r)!}$$ D2

Together: one block; apart: the gaps P1

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots,\ |x| < 1$$ P2

$$(a+bx)^n = a^n\left(1+\frac{b}{a}x\right)^n$$ P3

Order matters? Decide first W1.

No Pascal for $n = -\frac{1}{2}$ W2.

Always state the validity W2.

WHAT UNLOCKS

- "in a row", "a code" → order matters: $n!$, ${}^nP_r$.
- "a team", "a committee" → $\binom{n}{r}$ for each group, multiplied.
- "at least one" → the total minus the number with none.
- a negative or fractional power → the infinite series, then $|x| < \ldots$

WORKED EXAMPLE ①

Find the number of ways a gold, a silver and a bronze medal can be given to three of seven runners. no GDC

order matters: ${}^7P_3 = 7 \times 6 \times 5$ (M1)

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Method

A permutation

① Does order matter? Here, yes.
② r factors, from n down.
③ No division by $r!$.

WORKED EXAMPLE ②

Find the coefficient of x^2 in the expansion of $(1-2x)^{-3}$. no GDC

$$\frac{(-3)(-4)}{2!}(-2x)^2 = 6 \times 4x^2$$ (M1)

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Method

One coefficient, $n \notin \mathbb{N}$

① $\frac{n(n-1)\cdots}{r!}$ with $n = -3$.
② $(-2x)^r$ in brackets.
③ Only the number.

Valid for $|x| < \frac{1}{2}$.

FLUENCY

1. $6!$ no GDC

2. ${}^8P_2$ no GDC

3. $\binom{9}{3}$ no GDC

4. Four people in a row: how many orders? no GDC

5. $(1+x)^{-1}$, first three terms no GDC

6. $(1+x)^{1/2}$, first three terms no GDC

REASONING

●●● **Question 1.** Four-digit codes are made from the digits 1 to 9, with no digit repeated. [no GDC]

(a) Find the number of even codes. [2]

●●● **Question 2.** Five friends sit in a row, but two of them refuse to sit next to each other. [no GDC]

(a) Find the number of possible rows. [3]

●●● **Question 3.** Consider $(1+2x)^{-2}$. [no GDC]

(a) Expand up to and including the term in x^3, and state the values of x for which the expansion is valid. [4]

PROBLEMS

●●● **Question 4.** Consider $f(x) = \sqrt{9+x}$. [no GDC]

(a) Expand $f(x)$ up to and including the term in x^2, and state where the expansion is valid. [4]

(b) Hence estimate $\sqrt{9.18}$. [2]

●●● **Question 5.** A team of five is chosen from four girls and six boys, with at least one girl; then one member of the team is chosen as captain. [no GDC]

(a) Find the number of possible teams with a captain. [4]

ANSWERS

1 720

2 56

3 84

4 24

5 $1 - x + x^2$

6 $1 + \frac{x}{2} - \frac{x^2}{8}$

Q1 last digit $2, 4, 6$ or 8: $4 \times 8 \times 7 \times 6 = 1344$

Q2 $5! - 2 \times 4! = 120 - 48 = 72$

Q3 $1 - 4x + 12x^2 - 32x^3$, valid for $-\frac{1}{2} < x < \frac{1}{2}$

Q4 $3 + \frac{x}{6} - \frac{x^2}{216}$, valid for $-9 < x < 9$; $x = 0.18$: 3.02985

Q5 $\left(\binom{10}{5} - \binom{6}{5}\right) \times 5 = (252 - 6) \times 5 = 1230$

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