Marked example 1

Paper 1 style · No calculator

1 7 marks AHL 1.14 ●●● Demanding

Let $c = \cos\theta$ and $s = \sin\theta$.

(a) By considering $(c + \mathrm{i}s)^3$, show that $\cos 3\theta = 4c^3 - 3c$. **[4]**

(b) Hence solve the equation $8x^3 - 6x - 1 = 0$, giving your answers in the form $\cos\alpha$, where $0 \leq \alpha \leq \pi$. **[3]**

MARKSCHEME (a)

$(c + \mathrm{i}s)^3 = \cos 3\theta + \mathrm{i}\sin 3\theta$ **M1**

$= c^3 + 3c^2(\mathrm{i}s) + 3c(\mathrm{i}s)^2 + (\mathrm{i}s)^3$ **M1**

$= c^3 - 3cs^2 + \mathrm{i}\left(3c^2 s - s^3\right)$ **A1**

real parts: $\cos 3\theta = c^3 - 3c\left(1 - c^2\right)$ **A1**

$= 4c^3 - 3c$ **AG**

Total 4 marks

Note De Moivre, then the binomial expansion; $s^2 = 1 - c^2$ must be seen.

MARKSCHEME (b)

$x = \cos\theta$: $2\cos 3\theta - 1 = 0$, $\cos 3\theta = \frac{1}{2}$ **M1**

$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$ **(A1)**

$x = \cos\frac{\pi}{9}, \cos\frac{5\pi}{9}, \cos\frac{7\pi}{9}$ **A1**

Total 3 marks

Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software.
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