Exercise 1

Paper 1 style · No calculator

1 7 marks AHL 1.14 ●●● Demanding

Let $c = \cos\theta$ and $s = \sin\theta$.

(a) By considering $(c + \mathrm{i}s)^3$, show that $\sin 3\theta = 3s - 4s^3$. **[4]**

(b) Hence solve the equation $8x^3 - 6x + 1 = 0$, giving your answers in the form $\sin\alpha$, where $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$. **[3]**

The markscheme is on the next page: try the question first. Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization.
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MARKSCHEME (a)

$(c + \mathrm{i}s)^3 = \cos 3\theta + \mathrm{i}\sin 3\theta$ **M1**

$= c^3 + 3c^2(\mathrm{i}s) + 3c(\mathrm{i}s)^2 + (\mathrm{i}s)^3$ **M1**

$= c^3 - 3cs^2 + \mathrm{i}\left(3c^2 s - s^3\right)$ **A1**

imaginary parts: $\sin 3\theta = 3\left(1 - s^2\right)s - s^3$ **A1**

$= 3s - 4s^3$ **AG**

Total 4 marks

MARKSCHEME (b)

$x = \sin\theta$: $-2\sin 3\theta + 1 = 0$, $\sin 3\theta = \frac{1}{2}$ **M1**

$3\theta = -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$ **(A1)**

$x = \sin\left(-\frac{7\pi}{18}\right), \sin\frac{\pi}{18}, \sin\frac{5\pi}{18}$ **A1**

Total 3 marks

Note $8x^3 - 6x = -2\left(3s - 4s^3\right)$.