Marked example 1

Paper 1 style · No calculator

1 7 marks AHL 1.15 ●●○ Typical

Prove by mathematical induction that

$$\sum_{r=1}^{n} r \times 2^r = (n-1)2^{n+1} + 2 \quad \text{for all } n \in \mathbb{Z}^+.$$

MARKSCHEME

$n = 1$: $1 \times 2 = 2$ and $0 \times 4 + 2 = 2$, true	A1
assume true for $n = k$: $\sum_{r=1}^{k} r2^r = (k-1)2^{k+1} + 2$, for some $k \in \mathbb{Z}^+$	M1
$\sum_{r=1}^{k+1} r2^r = (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$	M1
$= ((k-1) + (k+1))\, 2^{k+1} + 2$	A1
$= 2k \times 2^{k+1} + 2$	A1
$= k \times 2^{k+2} + 2 = ((k+1) - 1)\, 2^{(k+1)+1} + 2$	A1
true for $n = 1$, and true for $n = k$ implies true for $n = k+1$; hence true for all $n \in \mathbb{Z}^+$	R1

Total 7 marks

Note The R1 is awarded only if the three method or accuracy marks before it were earned.

Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software.
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