Marked example 2

Paper 1 style · No calculator

2 7 marks AHL 1.15 ●●○ Typical

Prove by mathematical induction that $3^{2n+1} + 2^{n+2}$ is divisible by 7 for all $n \in \mathbb{Z}^+$.

MARKSCHEME

$n = 1$: $27 + 8 = 35 = 7 \times 5$ **A1**

assume $3^{2k+1} + 2^{k+2} = 7m,\ m \in \mathbb{Z}$, for some $k \in \mathbb{Z}^+$ **M1**

consider $3^{2(k+1)+1} + 2^{(k+1)+2} = 3^{2k+3} + 2^{k+3}$ **M1**

$= 9 \times 3^{2k+1} + 2 \times 2^{k+2}$ **A1**

$= 9\left(3^{2k+1} + 2^{k+2}\right) - 7 \times 2^{k+2}$ **A1**

$= 7\left(9m - 2^{k+2}\right)$, a multiple of 7 **A1**

true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$; hence true for all $n \in \mathbb{Z}^+$ **R1**

Total 7 marks

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