Marked example 3

Paper 2 style · Graphic display calculator

3 7 marks AHL 1.15 ●●○ Typical

A sequence is defined by $u_1 = 3$ and $u_{n+1} = 2u_n - 1$ for $n \in \mathbb{Z}^+$.

(a) Find u_2, u_3, u_4 and u_5. **[1]**

(b) Suggest an expression for u_n in terms of n. **[1]**

(c) Prove your answer to part (b) by mathematical induction. **[5]**

MARKSCHEME (a)

5, 9, 17, 33 — A1

Total 1 mark

MARKSCHEME (b)

$u_n = 2^n + 1$ — A1

Total 1 mark

MARKSCHEME (c)

$n = 1$: $2^1 + 1 = 3 = u_1$ — A1

assume $u_k = 2^k + 1$ for some $k \in \mathbb{Z}^+$ — M1

$u_{k+1} = 2u_k - 1 = 2\left(2^k + 1\right) - 1$ — M1

$= 2^{k+1} + 1$ — A1

true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$; hence true for all $n \in \mathbb{Z}^+$ — R1

Total 5 marks

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