Exercise 1

Paper 1 style · No calculator

1 7 marks AHL 1.15

●●○ Typical

Prove by mathematical induction that

$$\sum_{r=1}^{n} r \times 3^r = \frac{(2n-1)3^{n+1}+3}{4} \quad \text{for all } n \in \mathbb{Z}^+.$$

The markscheme is on the next page: try the question first. Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization.
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MARKSCHEME

$n = 1$: $1 \times 3 = 3$ and $\frac{1\times 9+3}{4} = 3$, true — **A1**

assume true for $n = k$: $\sum_{r=1}^{k} r3^r = \frac{(2k-1)3^{k+1}+3}{4}$, for some $k \in \mathbb{Z}^+$ — **M1**

$\sum_{r=1}^{k+1} r3^r = \frac{(2k-1)3^{k+1}+3}{4} + (k+1)3^{k+1}$ — **M1**

$= \frac{(2k-1+4k+4)3^{k+1}+3}{4}$ — **A1**

$= \frac{(6k+3)3^{k+1}+3}{4}$ — **A1**

$= \frac{(2k+1)3^{k+2}+3}{4} = \frac{(2(k+1)-1)3^{(k+1)+1}+3}{4}$ — **A1**

true for $n = 1$, and true for $n = k$ implies true for $n = k+1$; hence true for all $n \in \mathbb{Z}^+$ — **R1**

Total 7 marks