Exercise 2

Paper 1 style · No calculator

2 7 marks AHL 1.15 ●●○ Typical

Prove by mathematical induction that $4^{n+1} + 5^{2n-1}$ is divisible by 21 for all $n \in \mathbb{Z}^+$.

The markscheme is on the next page: try the question first. Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization.
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MARKSCHEME

$n = 1$: $16 + 5 = 21$ **A1**

assume $4^{k+1} + 5^{2k-1} = 21m,\ m \in \mathbb{Z}$, for some $k \in \mathbb{Z}^+$ **M1**

consider $4^{(k+1)+1} + 5^{2(k+1)-1} = 4^{k+2} + 5^{2k+1}$ **M1**

$= 4 \times 4^{k+1} + 25 \times 5^{2k-1}$ **A1**

$= 4\left(4^{k+1} + 5^{2k-1}\right) + 21 \times 5^{2k-1}$ **A1**

$= 21\left(4m + 5^{2k-1}\right)$, a multiple of 21 **A1**

true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$; hence true for all $n \in \mathbb{Z}^+$ **R1**

Total 7 marks