Exercise 3

Paper 2 style · Graphic display calculator

3 7 marks AHL 1.15 ●●○ Typical

A sequence is defined by $u_1 = 2$ and $u_{n+1} = 3u_n - 2$ for $n \in \mathbb{Z}^+$.

(a) Find u_2, u_3, u_4 and u_5. **[1]**

(b) Suggest an expression for u_n in terms of n. **[1]**

(c) Prove your answer to part (b) by mathematical induction. **[5]**

The markscheme is on the next page: try the question first. Written and verified by a practising IB mathematics teacher, produced with AI assistance; solved twice and recomputed by software. Not affiliated with or endorsed by the International Baccalaureate Organization.
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MARKSCHEME (a)

4, 10, 28, 82 — A1

Total 1 mark

MARKSCHEME (b)

$u_n = 3^{n-1} + 1$ — A1

Total 1 mark

MARKSCHEME (c)

$n = 1$: $3^0 + 1 = 2 = u_1$ — A1

assume $u_k = 3^{k-1} + 1$ for some $k \in \mathbb{Z}^+$ — M1

$u_{k+1} = 3u_k - 2 = 3\left(3^{k-1} + 1\right) - 2$ — M1

$= 3^k + 1$ — A1

true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$; hence true for all $n \in \mathbb{Z}^+$ — R1

Total 5 marks