Averages and spread

Two numbers summarise a set of data: where it sits, and how far it spreads. The mean, the median and the mode answer the first question; the range, the interquartile range and the standard deviation answer the second. Adding a constant moves the data; multiplying stretches them too.

I MEASURES OF CENTRAL TENDENCY

SL 4.3 · mean, median, mode, from a list, a table, grouped data

I.1 Mean, median and mode of a list

Definition 1 *mean, median, mode*

Mean $\bar{x} = \dfrac{\sum x}{n}$, or $\dfrac{\sum fx}{\sum f}$ from a frequency table. **Median**: the middle value of the ordered data, the $\frac{n+1}{2}$th. **Mode**: the most frequent value; for grouped data, the **modal class**.

Example 1

Find the mean, the median and the mode of 3, 8, 5, 12, 5, 9, 7.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Mean: total over number	$\dfrac{49}{7} = 7$
Step 2 Order, take the $\frac{n+1}{2}$th	3, 5, 5, 7, 8, 9, 12: the 4th, 7
Step 3 Mode: the most frequent	5

APPLY

1 Mean, median, mode of 2, 4, 4, 9, 11, 6.

2 For 10 ordered data, which position is the median?

I.2 The mean from a frequency table

Watch out 1

The mean of a frequency table divides by $\sum f$, the number of data, not by the number of rows.

Example 2

The numbers of brothers and sisters of 20 students: $x = 0, 1, 2, 3$ with frequencies 6, 9, 3, 2. Find the mean and the median.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $\sum f$ and $\sum fx$	20 and $0 + 9 + 6 + 6 = 21$
Step 2 Mean	$\frac{21}{20} = 1.05$
Step 3 Median: the 10th and 11th values	cumulative 6, 15: both are 1, median 1

APPLY

3 $x = 1, 2, 3, 4$ with frequencies 5, 8, 4, 3: mean and median.

I.3 An unknown frequency

Example 3

$x = 1, 2, 3$ with frequencies 4, k, 6. The mean is 2.1. Find k.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the mean with k	$\frac{4 + 2k + 18}{10 + k} = 2.1$
Step 2 Multiply out	$22 + 2k = 21 + 2.1k$
Step 3 Solve	$k = 10$

APPLY

4 $x = 0, 1, 2$ with frequencies 2, 3, k; the mean is 1.3. Find k.

I.4 Estimating the mean of grouped data

Property 1 *grouped data*

For grouped data the mean is **estimated** with the mid-interval values: $\bar{x} \approx \frac{\sum fx}{\sum f}$.
The GDC gives it from the midpoints in one list, the frequencies in another.

Example 4

Heights h cm of 20 students: $150 < h \leq 160$: 4; $160 < h \leq 170$: 10; $170 < h \leq 180$: 6. Estimate the mean and give the modal class.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Mid-interval values	$155,\ 165,\ 175$
Step 2 $\sum fx$	$620 + 1650 + 1050 = 3320$
Step 3 Divide by $\sum f$, say "estimate"	$\bar{x} \approx \dfrac{3320}{20} = 166$ cm
Step 4 Modal class	$160 < h \leq 170$

APPLY

5 $0 < m \leq 4$: 6; $4 < m \leq 8$: 10; $8 < m \leq 12$: 4. Estimate the mean.

II.1 Range and standard deviation

Definition 2 *standard deviation*

Variance $\sigma^2 = \dfrac{\sum(x-\bar{x})^2}{n}$, **standard deviation** $\sigma = \sqrt{\sigma^2}$, in the units of the data; at SL, found with the GDC. **Range** max − min; **interquartile range** $Q_3 - Q_1$.

Example 5

For 4, 6, 7, 9, 14, find the range, the mean and the standard deviation.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Range	$14 - 4 = 10$
Step 2 Mean	$\dfrac{40}{5} = 8$
Step 3 Standard deviation (GDC, σ)	$\sigma = \sqrt{\dfrac{16+4+1+1+36}{5}} = \sqrt{11.6} = 3.41$

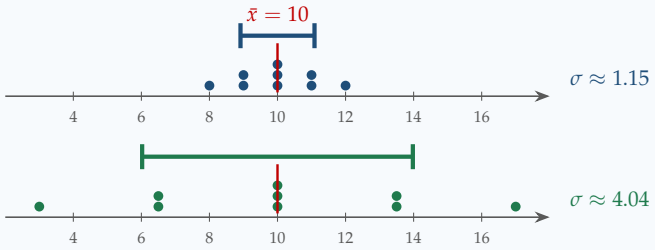


Figure 1 — Two sets of nine data with the same mean, 10: close to it, $\sigma \approx 1.15$; spread out, $\sigma \approx 4.04$.

APPLY

1 σ of 2, 4, 6, 8.

2 σ of 5, 5, 5, 5.

II.2 Adding or removing a value

Example 6

The mean of 12 numbers is 20. A 13th number is added and the mean becomes 21. Find the 13th number.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Total before: $n\bar{x}$	$12 \times 20 = 240$
Step 2 Total after	$13 \times 21 = 273$
Step 3 Difference	$273 - 240 = 33$

APPLY

3 The mean of 5 numbers is 9. The number 13 is removed. The new mean?

II.3 Changing every value

Property 2 *changing every value*

Every value $+b$: averages and quartiles $+b$; range, IQR, standard deviation unchanged. Every value $\times a$ $(a > 0)$: every average and every measure of spread $\times a$; the variance $\times a^2$.

Watch out 2

Adding 5 to every value does not change the standard deviation: the data move, they do not spread.

Example 7

A data set has mean 40 and standard deviation 6. Each value is multiplied by 1.5, then 10 is subtracted. Find the new mean, standard deviation and variance.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Mean: $a\bar{x} + b$	$1.5 \times 40 - 10 = 50$
Step 2 Standard deviation: $a\sigma$ only	$1.5 \times 6 = 9$
Step 3 Variance	$9^2 = 81$

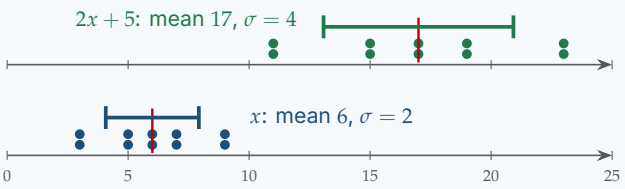


Figure 2 — The data x and $2x + 5$: the mean goes from 6 to 17, the standard deviation from 2 to 4 — the rule of Example 7.

APPLY

4 Mean 7, $\sigma = 2$; each value $+3$: the new mean and σ.

5 $\sigma = 2$; each value $\times 4$: the new variance.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A From a frequency table

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 5]

The number of pets owned by each student in a class is shown below.

pets	0	1	2	3	4
frequency	3	7	k	5	1

The mean number of pets is 1.8.

(a) Find the value of k. [3]

(b) Find the median number of pets. [1]

(c) Write down the mode. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the mean with k	$\dfrac{0+7+2k+15+4}{16+k} = 1.8$	M1A1
	M1 for $\frac{\sum fx}{\sum f}$; A1 for the correct equation.	
Step 2 Solve	$k = 14$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Median: the middle position, cumulative frequencies	2	A1
	30 students: the 15th and 16th values are both 2.	

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The largest frequency	2	A1

B Changing the data

EXAM-STYLE QUESTION 2 — Paper 1 · no calculator · [Maximum mark: 4]

The mean of 10 numbers is 12 and their standard deviation is 3.

(a) An eleventh number is added and the mean becomes 12.5. Find the eleventh number. [2]

(b) Instead, each of the original 10 numbers is multiplied by 2 and then 5 is added. Write down the new mean and the new standard deviation. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Totals before and after: $n\bar{x}$	$11 \times 12.5 - 10 \times 12 = 137.5 - 120$ *Totals first: the sums, not the means, add.*	(M1)
Step 2 Difference	17.5	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Mean: $a\bar{x} + b$	mean $2 \times 12 + 5 = 29$	A1
Step 2 Standard deviation: $a\sigma$	standard deviation $2 \times 3 = 6$ *Adding 5 does not change the spread.*	A1

C Grouped data on the GDC (Paper 2)

EXAM-STYLE QUESTION 3 Paper 2 · calculator · [Maximum mark: 5]

The times, t minutes, that 50 customers waited at a counter are grouped below.

t	$0 < t \leq 10$	$10 < t \leq 20$	$20 < t \leq 30$	$30 < t \leq 40$	$40 < t \leq 50$
frequency	5	12	18	10	5

(a) Write down the modal class. [1]
(b) Find an estimate of the mean waiting time. [2]
(c) Find an estimate of the standard deviation of the waiting times. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The largest frequency	$20 < t \leq 30$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Mid-interval values	mid-interval values 5, 15, 25, 35, 45	(M1)
Step 2 Estimate on the GDC	24.6 minutes	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 σ on the GDC, same lists	11.1 minutes (11.1283 . . .)	A2

IV WHICH METHOD? READ THE QUESTION

4.3 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"find the mean" of a frequency table	$\sum fx$ and $\sum f$	$\bar{x} = \frac{\sum fx}{\sum f}$ D1
an unknown frequency, the mean given	an equation in k	$\frac{\sum fx}{\sum f} = \bar{x}$, solve D1
"the median" of n data	the $\frac{n+1}{2}$th value	cumulative frequencies to find it D1
"estimate the mean" of grouped data	mid-interval values	say "estimate"; the GDC with midpoints and frequencies P1
"standard deviation"	the GDC, σ	the population value σ, to 3 s.f. D2
every value $+b$, or $\times a$	which measures move	mean $a\bar{x} + b$; standard deviation $a\sigma$ P2
a value added or removed	the totals	$n\bar{x}$ before and after

In the exam 1 *command terms*

Estimate: from grouped data, the result is approximate. **Standard deviation**: σ, from the GDC. **Write down**: no working, but units if there are any.

V ON THE GDC: PAPER 2

SL 4.3 · NumWorks · every calculator move of this topic

1 Mean and standard deviation from a table

Statistics → V1: values, N1: frequencies → Stats

Midpoints in V1, frequencies in N1, then Stats. **On paper:** "estimate of the mean 24.6, standard deviation 11.1".

STATISTICS · STATS

```
V1: 5,15,25,35,45 N1:
5,12,18,10,5
mean 24.6
sigma 11.12834
```

ANSWERS TO THE APPLY QUESTIONS

I.1 6; 5; 4

I.2 the 5.5th: the mean of the 5th and 6th

I.3 2.25; 2

I.4 $k = 5$

I.5 5.6

II.1 $\sqrt{5} = 2.24$

II.2 0

II.3 8

II.4 10; 2

II.5 64

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