Averages and spread

Skills sheet · SL AA 4.3

KEY FACTS

$\bar{x} = \dfrac{\sum fx}{\sum f}$; median the $\frac{n+1}{2}$th D1

Grouped data: mid-interval values, an estimate P1

σ: the typical distance to the mean, from the GDC D2

$ax + b$: mean $a\bar{x} + b$, $\sigma \to a\sigma$, variance $\times a^2$ P2

Divide by $\sum f$, not the rows W1.

Adding b: no change to σ W2.

A value added: work with totals.

WHAT UNLOCKS

- a frequency table $\to$ $\sum fx$ and $\sum f$.
- an unknown frequency $\to$ an equation from the mean.
- grouped data $\to$ midpoints, the GDC, the word "estimate".
- every value changed $\to$ which measures move, and by how much.

WORKED EXAMPLE ①

$x = 1, 2, 3$ with frequencies 4, 5, 1. Find the mean. no GDC

$\frac{4+10+3}{10}$ (M1)

1.7 A1

Method

The mean of a table

① Multiply each value by its frequency.
② Add, divide by $\sum f$.
③ Check it lies between the extremes.

WORKED EXAMPLE ②

Mean 20, $\sigma = 5$. Each value is multiplied by 3, then 4 is subtracted. New mean and σ? no GDC

mean $3 \times 20 - 4 = 56$ A1

$\sigma = 3 \times 5 = 15$ A1

Method

Changing every value

① The mean follows both steps.
② σ follows the stretch only.
③ Variance: a^2.

Subtracting does not shrink the spread.

FLUENCY

●○○ **1.** Mean of 4, 7, 7, 10, 12 no GDC

●○○ **2.** Median of 4, 7, 7, 10, 12 no GDC

●○○ **3.** Mode of 4, 7, 7, 10, 12 no GDC

●○○ **4.** Range of 4, 7, 7, 10, 12 no GDC

●○○ **5.** Mean of $x = 1, 2, 3$, frequencies 2, 5, 3 no GDC

●○○ **6.** $\sigma = 3$: the variance no GDC

REASONING

●●● **Question 1.** The mean of 6 numbers is 9. One number is removed and the mean of the other 5 is 8. [no GDC]

(a) Find the number removed. [2]

●●● **Question 2.** A set of data has standard deviation 2.5. Each value is increased by 7, then doubled. [no GDC]

(a) Find the new standard deviation and the new variance. [2]

●●● **Question 3.** 20 values are grouped: $0 < x \leq 10$: 4; $10 < x \leq 20$: 10; $20 < x \leq 30$: 6. [no GDC]

(a) Write down the modal class. [1]

(b) Find an estimate of the mean. [2]

PROBLEMS

●●● **Question 4.** The values 1, 2, 3, 4 have frequencies 3, a, b, 2. There are 15 values and their mean is 2.4. [no GDC]

(a) Find a and b. [4]

●●● **Question 5.** Test marks have mean 52 and standard deviation 10. Each mark x is scaled to $ax + b$, $a > 0$, so that the new mean is 60 and the new standard deviation 12. [no GDC]

(a) Find a and b. [3]

ANSWERS

1 8

2 7

3 7

4 8

5 $\frac{2+10+9}{10} = 2.1$

6 9

Q1 $54 - 40 = 14$

Q2 $\sigma = 5$, variance 25

Q3 (a) $10 < x \leq 20$ (b) $\frac{4\times5+10\times15+6\times25}{20} = 16$

Q4 $a + b = 10,\ 3 + 2a + 3b + 8 = 36$: $a = 5, b = 5$

Q5 $a = \frac{12}{10} = 1.2;\ 1.2 \times 52 + b = 60,\ b = -2.4$

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