Discrete and binomial distributions

SL AA · Statistics and probability · 4.7 · 4.8

A random variable gives a number to each outcome; its distribution says how likely each number is. The expected value is the balance point of that distribution. When the same independent trial is repeated n times, the number of successes follows the binomial distribution.

I DISCRETE RANDOM VARIABLES

SL 4.7 · distributions, $\sum P = 1$, $E(X)$, fair games

I.1 A probability distribution

Definition 1 *probability distribution*

A **discrete random variable** X takes values x_i with probabilities $P(X = x_i)$, $\sum P(X = x_i) = 1$. Its **expected value** is $E(X) = \sum x\,P(X = x)$: the mean of X over many repetitions.

Example 1

X takes the values 0, 1, 2, 3 with probabilities 0.15, $2k$, 0.25, k. Find k and $P(X \leq 1)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 The probabilities add up to 1	$0.4 + 3k = 1$
Step 2 Solve	$k = 0.2$
Step 3 Add the probabilities wanted	$P(X \leq 1) = 0.15 + 0.4 = 0.55$

APPLY

1 $x = 1, 2, 3$ with probabilities 0.35, k, 0.25: find k.

2 $P(X = x) = kx^2$ for $x = 1, 2, 3$: find k.

I.2 The expected value

Example 2

Find $E(X)$ for the distribution of Example 1.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Multiply each value by its probability	$0 \times 0.15 + 1 \times 0.4 + 2 \times 0.25 + 3 \times 0.2$
Step 2 Add	$E(X) = 1.5$

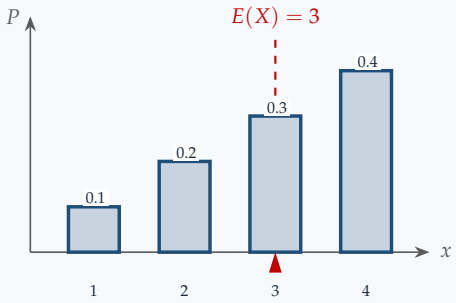


Figure 1 — Exam-style question 1's distribution, 0.1, 0.2, 0.3, 0.4 on 1 to 4: the bars balance at $E(X) = 3$.

APPLY

3 $x = 2, 5, 10$ with probabilities 0.5, 0.3, 0.2: $E(X)$.

4 $x = -1$, 3 with probabilities 0.6, 0.4: $E(X)$.

I.3 Fair games

Property 1 *fair games*

Let X be the gain of the player (winnings minus stake). The game is **fair** when $E(X) = 0$.

Watch out 1

$E(X)$ need not be a value X can take, nor the most likely value. Check $\sum P = 1$ first.

Example 3

A player pays 5 euros to roll a fair die: a six wins 12 euros, a 4 or a 5 wins 3 euros, otherwise nothing. Is the game fair? What stake would make it fair?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Expected winnings	$12 \times \frac{1}{6} + 3 \times \frac{2}{6} = 3$
Step 2 Expected gain: minus the stake	$3 - 5 = -2 \neq 0$: not fair
Step 3 Fair stake: the expected winnings	3 euros

APPLY

5 Stake 2 euros, win 7 euros with probability 0.25: the expected gain.

6 Stake 1 euro, win probability 0.2: the prize for a fair game.

II THE BINOMIAL DISTRIBUTION

SL 4.8 · $X \sim B(n, p)$, $P(X = r)$, $P(X \leq r)$, $E(X) = np$, $\text{Var}(X) = np(1-p)$

II.1 Recognising a binomial

Definition 2 *binomial distribution*

The same two-outcome trial repeated n times **independently**, with the same probability p of success: the number of successes follows $X \sim B(n, p)$. The GDC gives $P(X = r)$ and $P(X \leq r)$.

Example 4

A student guesses all 10 answers of a test with 4 options each. Let X be the number of correct answers. Explain why X is binomial and give its parameters.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Fixed number of trials	$n = 10$ questions
Step 2 Two outcomes, same p	right or wrong, $p = \frac{1}{4}$ each time
Step 3 Independent	each guess does not affect the others
Step 4 Write the distribution	$X \sim B(10, 0.25)$

APPLY

1 Two cards are drawn without replacement; X the number of aces. Binomial?

2 A die is thrown 30 times; X the number of sixes.

II.2 $P(X = r)$

Example 5

$X \sim B(10, 0.25)$. Find $P(X = 3)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the probability statement	$P(X = 3)$
Step 2 Binomial pdf on the GDC: $n = 10$, $p = 0.25$, $r = 3$	$0.250282\ldots$
Step 3 Answer, 3 s.f.	0.250

APPLY

3 $B(6, 0.5)$: $P(X = 2)$.

4 $B(15, 0.1)$: $P(X = 0)$.

II.3 At least, at most, more than

Watch out 2

"At least 5": $P(X \geq 5) = 1 - P(X \leq 4)$. "More than 4": the same. "Fewer than 3": $P(X \leq 2)$. Write the statement before the number.

Example 6

$X \sim B(10, 0.25)$. Find $P(X \leq 2)$ and the probability of at least 4 successes.

WHAT YOU DO	WHAT YOU WRITE
Step 1 At most 2: cumulative	$P(X \leq 2) = 0.526$
Step 2 At least 4: the complement	$P(X \geq 4) = 1 - P(X \leq 3)$
Step 3 Compute	0.224

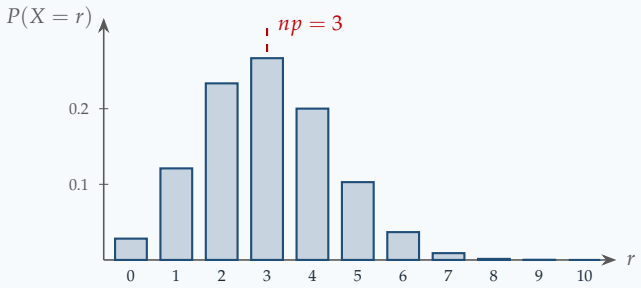


Figure 2 — $B(10, 0.3)$: the bars of $P(X = r)$, highest at $r = 3$, the mean $np = 3$.

APPLY

5 $B(8, 0.6)$: $P(X < 3)$.

6 $B(8, 0.6)$: $P(X > 6)$.

II.4 Mean and variance

Property 2 *mean and variance*

$E(X) = np$, $\text{Var}(X) = np(1-p)$; standard deviation $\sqrt{np(1-p)}$.

Example 7

$X \sim B(40, 0.15)$. Find the mean, the variance and the standard deviation of X.

WHAT YOU DO	WHAT YOU WRITE
Step 1 $E(X) = np$	$40 \times 0.15 = 6$
Step 2 $\text{Var}(X) = np(1-p)$	$6 \times 0.85 = 5.1$
Step 3 Square root	$\sigma = 2.26$

APPLY

7 $B(25, 0.4)$: mean and variance.

8 $X \sim B(48, p)$ and $E(X) = 12$: find p.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Discrete random variables

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 5]

The probability distribution of a discrete random variable X is

x	1	2	3	4
$P(X = x)$	0.1	k	0.3	$2k$

(a) Find the value of k. [2]
(b) Find $E(X)$. [2]
(c) Find $P(X \geq 3)$. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The probabilities add up to 1	$0.1 + k + 0.3 + 2k = 1$ *The probabilities add up to 1.*	(M1)
Step 2 Solve	$k = 0.2$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Multiply each value by its probability	$1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3 + 4 \times 0.4$	(M1)
Step 2 Add	$E(X) = 3$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Add the probabilities wanted	$0.3 + 0.4 = 0.7$	A1

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 5]

In a game, a player pays k euros to play. The player wins 10 euros with probability 0.2, 3 euros with probability 0.3, and nothing otherwise.

(a) Find the value of k for which the game is fair. [3]

(b) The organiser sets $k = 4$. Find the expected loss of a player per game. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Expected winnings	$E(\text{winnings}) = 10 \times 0.2 + 3 \times 0.3 = 2.9$ *Or the gain: $E(X) = 2.9 - k$.*	(M1)(A1)
Step 2 Fair: the stake equals them	fair: $k = 2.9$ *$E(\text{gain}) = 0$.*	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Expected gain: minus the stake	$E(\text{gain}) = 2.9 - 4 = -1.1$	(M1)
Step 2 Say it as a loss	expected loss 1.10 euros	A1

B The binomial distribution (Paper 2)

EXAM-STYLE QUESTION 3 Paper 2 · calculator · [Maximum mark: 6]

A spinner lands on red with probability 0.3. It is spun 12 times. Let X be the number of times it lands on red.

(a) Find $P(X = 4)$. [2]

(b) Find the probability that it lands on red at least 5 times. [2]

(c) Write down $E(X)$ and find $\mathrm{Var}(X)$. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the distribution	$X \sim B(12,\ 0.3)$	(M1)
Step 2 Binomial pdf	$0.231\ \ (0.231139\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 At least 5: the complement	$P(X \geq 5) = 1 - P(X \leq 4)$ *"At least 5": 4 is the last value excluded.*	(M1)
Step 2 Compute	$0.276\ \ (0.276344\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $E(X) = np$	$E(X) = 3.6$	A1
Step 2 $\mathrm{Var}(X) = np(1-p)$	$\mathrm{Var}(X) = 12 \times 0.3 \times 0.7 = 2.52$	A1

EXAM-STYLE QUESTION 4

Paper 2 · calculator · [Maximum mark: 7]

On any school day, the probability that a bus is late is 0.15, independently of other days. Consider 20 school days.

(a) Write down the expected number of days on which the bus is late. [1]

(b) Find the probability that the bus is late on exactly 2 days. [2]

(c) Find the probability that the bus is late on more than 4 days. [2]

(d) Find the least number of days n for which the probability that the bus is late at least once is greater than 0.99. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $E(X) = np$	$20 \times 0.15 = 3$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the distribution, binomial pdf	$X \sim B(20,\ 0.15)$	(M1)
Step 2 Compute	$0.229\ \ (0.229338\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 More than 4: the complement	$P(X > 4) = 1 - P(X \leq 4)$	(M1)
Step 2 Compute	$0.170\ \ (0.170153\ldots)$	A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the inequality in n	$1 - 0.85^n > 0.99$ *Or a table of values on the GDC.*	(M1)
Step 2 Solve on the GDC, next integer	$n = 29$ *$n > 28.3$: the least whole number is 29.*	A1

IV WHICH METHOD? READ THE QUESTION

4.8 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
an unknown in a table of probabilities	$\sum P = 1$	an equation in k D1
"expected value", "on average"	$\sum x\,P(X = x)$	multiply each value by its probability, add D1
"the game is fair"	$E(\text{gain}) = 0$	gain = winnings − stake P1
"n independent trials, probability p"	$X \sim B(n,\, p)$	write the distribution first D2
"exactly r"	$P(X = r)$	binomial pdf on the GDC D2
"at least", "more than", "at most"	$P(X \leq r)$ and complements	$P(X \geq r) = 1 - P(X \leq r - 1)$ W2
"mean", "variance" of a binomial	np, $np(1-p)$	no GDC needed P2

In the exam 1 *command terms*

Expected value: $E(X)$, a mean. **Fair**: expected gain 0. **State the distribution**: "$X \sim B(n,\, p)$" with the numbers. **At least**: $\geq$; **more than**: $>$; **at most**: $\leq$; **fewer than**: $<$.

V ON THE GDC: PAPER 2

SL 4.7 · 4.8 · NumWorks · every calculator move of this topic

1 P(X = r) for a binomial

Probability → Binomial → n, p → P(X = r)

Choose the binomial law, enter n and p, then $P(X = x)$. **On paper:** "$X \sim B(12,\, 0.3)$, $P(X = 4) = 0.231$".

PROBABILITY · BINOMIAL

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B(12, 0.3)
P(X=4) = 0.2311397
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2 P(X ≤ r), and its complement

Probability → Binomial → n, p → P(X ≤ r)

The GDC gives $P(X \leq r)$; take it from 1 for "at least". **On paper:** $P(X \geq 5) = 1 - P(X \leq 4) = 0.276$.

PROBABILITY · BINOMIAL

```
B(12, 0.3)
P(X<=4) = 0.7236555
1 - 0.7236555 = 0.2763445
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ANSWERS TO THE APPLY QUESTIONS

I.1 0.4

I.2 $\frac{1}{14}$

I.3 4.5

I.4 0.6

I.5 -0.25 euros

I.6 5 euros

II.1 no: p changes at the second draw

II.2 $B\left(30, \frac{1}{6}\right)$

II.3 0.234

II.4 0.206

II.5 0.0498

II.6 0.106

II.7 10; 6

II.8 0.25

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