Discrete and binomial distributions

Skills sheet · SL AA 4.7 · 4.8

KEY FACTS

$\sum P(X = x) = 1$; $E(X) = \sum x\,P(X = x)$ D1
Fair game: $E(\text{gain}) = 0$ P1
$X \sim B(n, p)$: n independent trials, same p D2
$E(X) = np$, $\text{Var}(X) = np(1-p)$ P2

At least r: $1 - P(X \leq r-1)$ W2.
$E(X)$ need not be a value of X W1.
State $B(n, p)$ first.

WHAT UNLOCKS

- an unknown probability → the probabilities add up to 1.
- "expected value" → each value times its probability.
- "fair" → expected gain 0.
- repeated independent trials → binomial; the GDC for $P(X = r)$, $P(X \leq r)$.

WORKED EXAMPLE ①

$P(X = 0, 1, 2) = 0.5, 0.3, 0.2$. **Find** $E(X)$. no GDC

$0 \times 0.5 + 1 \times 0.3 + 2 \times 0.2$ (M1)
0.7 A1

Method
An expected value
① Check $\sum P = 1$.
② Multiply each value by its probability.
③ Add.

WORKED EXAMPLE ②

$X \sim B(8, 0.5)$. **Find** $P(X = 4)$. with GDC

binomial pdf, $n = 8$, $p = 0.5$ (M1)
0.273 A1

Method
A binomial probability
① Write $X \sim B(n, p)$.
② The probability statement.
③ The GDC, to 3 s.f.
"At least": the complement.

FLUENCY

●○○ **1.** $P(X = 1, 2, 3) = 0.2, 0.5, k$: find k no GDC

●○○ **2.** Same distribution: $E(X)$ no GDC

●○○ **3.** $X \sim B(20, 0.25)$: $E(X)$ no GDC

●○○ **4.** $X \sim B(20, 0.25)$: $\text{Var}(X)$ no GDC

●○○ **5.** Write $P(X \geq 3)$ with $P(X \leq r)$ no GDC

●○○ **6.** Stake 2, win 6 with probability 0.25: expected gain no GDC

REASONING

●●● **Question 1.** A fair die is rolled 10 times; X is the number of sixes. [with GDC]

(a) Find $P(X = 2)$. [2]

(b) Find $P(X \leq 1)$. [1]

●●● **Question 2.** $P(X = x) = \dfrac{x}{10}$ for $x = 1, 2, 3, 4$. [no GDC]

(a) Find $E(X)$. [2]

●●● **Question 3.** $X \sim B(n, 0.2)$ and $E(X) = 6$. [no GDC]

(a) Find n and $\text{Var}(X)$. [3]

PROBLEMS

●●● **Question 4.** A ticket wins 20 euros with probability 0.05 and 5 euros with probability 0.2, and nothing otherwise. [no GDC]

(a) Find the price of a ticket that makes the game fair. [3]

●●● **Question 5.** A fair die is rolled n times. [with GDC]

(a) Find the least n for which the probability of at least one six is greater than 0.9. [3]

ANSWERS

1 0.3

2 $0.2 + 1 + 0.9 = 2.1$

3 5

4 3.75

5 $1 - P(X \leq 2)$

6 $1.5 - 2 = -0.5$

Q1 (a) 0.291 (b) 0.485

Q2 $\frac{1+4+9+16}{10} = 3$

Q3 $n = 30,\ \mathrm{Var}(X) = 4.8$

Q4 $20 \times 0.05 + 5 \times 0.2 = 2$ euros

Q5 $1 - \left(\frac{5}{6}\right)^n > 0.9,\ n > 12.6$: $n = 13$

Written and verified by a practising IB mathematics teacher, produced with AI assistance; every result recomputed by software, every figure computed from its function. Not affiliated with or endorsed by the International Baccalaureate Organization. © 2026 Hence