Probability

SL AA · Statistics and probability · 4.5 · 4.6 · 4.11

A probability is a proportion of outcomes. Everything on this topic comes from three ideas: count carefully (a Venn diagram, a table, a tree), do not count the overlap twice, and when a condition is given, shrink the universe to it.

I EVENTS AND PROBABILITIES

SL 4.5 · outcomes, sample space, $P(A')$, expected number

I.1 The probability of an event

Definition 1 *probability of an event*

Equally likely outcomes in the **sample space** U: $P(A) = \dfrac{n(A)}{n(U)}$. $0 \le P(A) \le 1$; the **complement** A', "not A", has $P(A') = 1 - P(A)$. In n trials, the **expected number** of times A occurs is $n\,P(A)$.

Example 1

A letter is chosen at random from the word PROBABILITY. Find $P(\text{B})$, $P(\text{a vowel})$ and $P(\text{not B})$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Count the sample space	11 letters
Step 2 Count each event	B: 2; vowels O, A, I, I: 4
Step 3 Divide	$P(\text{B}) = \dfrac{2}{11}$, $P(\text{vowel}) = \dfrac{4}{11}$
Step 4 Complement: $1 - P$	$P(\text{not B}) = \dfrac{9}{11}$

APPLY

1 A card from 1 to 20: $P(\text{a multiple of 3})$.

2 $P(A) = 0.42$: find $P(A')$.

I.2 The expected number

Watch out 1

An expected number need not be a whole number: $50 \times 0.03 = 1.5$ is fine. It is an average over many repetitions, not a prediction.

Example 2

The probability that an item is faulty is 0.04. How many faulty items are expected in a batch of 250?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Expected number: $n\,P(A)$	250×0.04
Step 2 Compute	10 items

APPLY

3 A fair die is thrown 90 times: the expected number of sixes.

4 $P(A) = 0.07$, 30 trials: the expected number.

II COMBINED EVENTS

SL 4.6 · $A \cup B$, $A \cap B$, Venn diagrams, tree diagrams, mutually exclusive events

II.1 The addition rule

Property 1 *the addition rule*

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$: the overlap is counted twice in $P(A) + P(B)$.
Mutually exclusive: $P(A \cap B) = 0$, then $P(A \cup B) = P(A) + P(B)$.

Example 3

$P(A) = 0.45$, $P(B) = 0.35$ and $P(A \cap B) = 0.15$. Find $P(A \cup B)$ and the probability of neither.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Addition rule	$0.45 + 0.35 - 0.15 = 0.65$
Step 2 Neither: the complement of $A \cup B$	$1 - 0.65 = 0.35$

APPLY

1 $P(A) = 0.6$, $P(B) = 0.25$, mutually exclusive: $P(A \cup B)$.

2 $P(A \cup B) = 0.9$, $P(A) = 0.5$, $P(B) = 0.6$: $P(A \cap B)$.

II.2 Venn diagrams

Example 4

Of 60 people, 35 drink tea, 28 drink coffee and 15 drink both. Complete a Venn diagram and find the probability that a person drinks exactly one of them.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Fill the overlap first	both: 15
Step 2 Then each part alone	tea only 20, coffee only 13
Step 3 Then outside	neither: $60 - 48 = 12$
Step 4 Exactly one	$\frac{20 + 13}{60} = \frac{11}{20}$

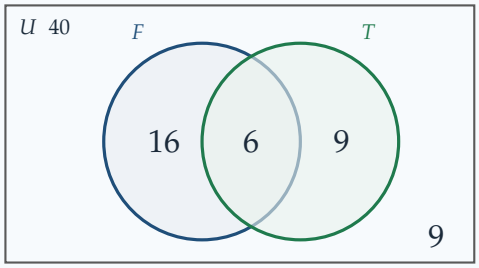


Figure 1 — Exam-style question 2: 40 students, 22 football, 15 tennis, 6 both — 16, 9 and 9 neither.

APPLY

3 30 students, 18 French, 12 Spanish, 5 both: how many take neither?

4 Same students: $P(\text{Spanish only})$.

II.3 Tree diagrams

Property 2 *tree diagrams*

Multiply along a branch for the probability of that path; add the paths that give the event.

Example 5

A spinner lands on red with probability 0.3. It is spun twice. Find $P(\text{both red})$ and $P(\text{exactly one red})$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Draw the tree: 0.3 and 0.7 at each stage	two stages, four paths
Step 2 Multiply along: both red	$0.3 \times 0.3 = 0.09$
Step 3 Add the two paths with one red	$0.3 \times 0.7 + 0.7 \times 0.3 = 0.42$

APPLY

5 A fair coin is thrown 3 times: $P(\text{three heads})$.

6 The spinner of Example 5: $P(\text{at least one red})$.

II.4 Without replacement

Example 6

A bag has 7 white and 2 black balls. Two are taken without replacement. Find $P(\text{both black})$ and $P(\text{one of each})$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Second stage: one ball fewer	denominators 9, then 8
Step 2 Both black	$\frac{2}{9} \times \frac{1}{8} = \frac{1}{36}$
Step 3 One of each: two paths	$\frac{7}{9} \times \frac{2}{8} + \frac{2}{9} \times \frac{7}{8} = \frac{7}{18}$

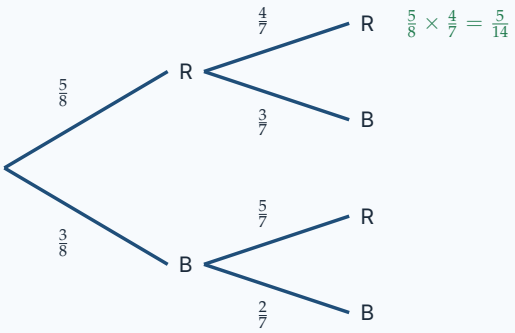


Figure 2 — Exam-style question 1: 5 red and 3 blue, without replacement; second-stage denominator 7.

APPLY

7 3 boys and 4 girls; two chosen at random: $P(\text{both girls})$.

III.1 Conditional probability

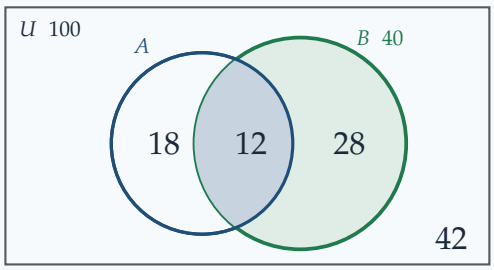
Definition 2 *conditional probability*

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$: B becomes the whole universe, and A is looked for inside it.

Example 7

Of 100 students, 45 walk to school, and 18 of those are late; 30 students are late in all. Find $P(\text{late} \mid \text{walk})$ and $P(\text{walk} \mid \text{late})$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Given "walk": only the 45 count	$P(\text{late} \mid \text{walk}) = \dfrac{18}{45} = 0.4$
Step 2 Given "late": only the 30 count	$P(\text{walk} \mid \text{late}) = \dfrac{18}{30} = 0.6$



$P(A \mid B) = \dfrac{12}{40} = 0.3$

$P(A) = \dfrac{30}{100} = 0.3$

Figure 3 — 100 people, 40 in B, 12 of them in A: given B, only the 40 count, $P(A \mid B) = 0.3$.

APPLY

1 $P(A \cap B) = 0.18$, $P(B) = 0.6$: $P(A \mid B)$.

2 Example 4's people: $P(\text{coffee} \mid \text{tea})$.

III.2 Conditional probability from a tree

Example 8

60 % of days are dry. A bus is late with probability 0.1 on a dry day and 0.3 on a wet day. Find $P(\text{late})$ and $P(\text{wet} \mid \text{late})$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Add the two late paths	$0.6 \times 0.1 + 0.4 \times 0.3 = 0.18$
Step 2 The path wet and late	$0.4 \times 0.3 = 0.12$
Step 3 Divide by the condition	$\dfrac{0.12}{0.18} = \dfrac{2}{3}$

APPLY

3 30 % of items come from machine A (5 % faulty), the rest from B (2 % faulty). $P(\text{faulty})$ and $P(A \mid \text{faulty})$.

III.3 Independent events

Definition 3 *independent events*

A and B are **independent** when one happening does not change the probability of the other: $P(A \mid B) = P(A)$, or $P(A \cap B) = P(A)\,P(B)$.

Watch out 2

Mutually exclusive is not independent: if A and B cannot both happen, knowing B tells you A did not. Compare $P(A \cap B)$ with $P(A)P(B)$ and conclude in words.

Example 9

$P(A) = 0.3$, $P(B) = 0.6$ and $P(A \cap B) = 0.18$. Are A and B independent?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Compute $P(A)P(B)$	$0.3 \times 0.6 = 0.18$
Step 2 Compare with $P(A \cap B)$	equal
Step 3 Conclude in words	A and B are independent

APPLY

4 $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.75$: independent?

5 Independent, $P(A) = 0.2$, $P(B) = 0.7$: $P(A \cup B)$.

IV EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Without replacement

EXAM-STYLE QUESTION 1 Paper 1 · no calculator · [Maximum mark: 6]

A bag contains 5 red counters and 3 blue counters. Two counters are taken at random, one after the other, without replacement.

(a) Find the probability that both counters are red. [2]

(b) Find the probability that the two counters are different colours. [2]

(c) Given that the counters are different colours, find the probability that the first one is red. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Multiply along the branch	$\frac{5}{8} \times \frac{4}{7}$	(M1)
Step 2 Simplify	$= \frac{20}{56} = \frac{5}{14}$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Add the two paths	$\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$ *Two paths: red then blue, blue then red.*	(M1)
Step 2 Simplify	$= \frac{30}{56} = \frac{15}{28}$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Conditional: divide by the condition	$\frac{15/56}{30/56}$ $P(\text{first red} \cap \text{different}) \div P(\text{different})$.	(M1)
Step 2 Simplify	$= \frac{1}{2}$	A1

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 6]

In a class of 40 students, 22 play football (F), 15 play tennis (T) and 6 play both.

(a) Find the number of students who play neither sport. [2]

(b) Find $P(F \cup T)$. [1]

(c) Find $P(T \mid F)$. [2]

(d) Determine whether F and T are independent. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 At least one: addition rule	$22 + 15 - 6 = 31$ play at least one	(M1)
Step 2 Neither: from the total	$40 - 31 = 9$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $P(F \cup T)$	$\dfrac{31}{40}$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Conditional: divide by $P(F)$	$\dfrac{P(T \cap F)}{P(F)} = \dfrac{6/40}{22/40}$	(M1)
Step 2 Simplify	$= \dfrac{6}{22} = \dfrac{3}{11}$	A1

WORKING (d)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Compare $P(F)P(T)$ with $P(F \cap T)$, conclude	$P(F)P(T) = \dfrac{22}{40} \times \dfrac{15}{40} \neq \dfrac{6}{40}$: not independent	R1

Or: $P(T \mid F) = \frac{3}{11} \neq P(T) = \frac{3}{8}$.

EXAM-STYLE QUESTION 3 Paper 1 · no calculator · [Maximum mark: 5]

Events A and B are such that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.7$.

(a) Find $P(A \cap B)$. [2]

(b) Show that A and B are independent. [2]

(c) Write down $P(A \mid B)$. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Addition rule with the unknown	$0.7 = 0.4 + 0.5 - P(A \cap B)$	(M1)
Step 2 Solve	$P(A \cap B) = 0.2$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Compute $P(A)P(B)$	$P(A)P(B) = 0.4 \times 0.5 = 0.2$	M1
Step 2 Compare, conclude	$= P(A \cap B)$, so independent	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Independent: $P(A \mid B) = P(A)$	0.4	A1

Independent: $P(A \mid B) = P(A)$.

C A tree in context (Paper 2)

EXAM-STYLE QUESTION 4 Paper 2 · calculator · [Maximum mark: 6]

In a town, 35 % of the people have been vaccinated (V). The probability of catching a cold (C) in winter is 0.25 for a vaccinated person and 0.6 for a person who has not been vaccinated.

(a) Find the probability that a person chosen at random catches a cold. [2]

(b) Given that a person caught a cold, find the probability that they were vaccinated. [2]

(c) In a sample of 400 people, find the expected number who catch a cold. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Add the two paths that give a cold	$0.35 \times 0.25 + 0.65 \times 0.6$	(M1)
Step 2 Compute	$= 0.4775$ *Accept* 0.478.	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Conditional: the path over the total	$\dfrac{0.35 \times 0.25}{0.4775}$	(M1)
Step 2 Compute	$= 0.183 \ \ (0.183246\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Expected number $n\,P$	400×0.4775	(M1)
Step 2 Compute	191 *Exactly* 191.	A1

V WHICH METHOD? READ THE QUESTION

4.11 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"how many expected"	$n\,P(A)$	the product; it may be a decimal D1
"not A"	the complement	$1 - P(A)$ D1
"A or B"	the addition rule	$P(A) + P(B) - P(A \cap B)$; a Venn diagram P1
two stages, "without replacement"	a tree diagram	multiply along, add across; the second stage changes P2
"given that"	a conditional probability	$\frac{P(A \cap B)}{P(\text{the condition})}$ D2
"are they independent?"	$P(A \cap B)$ and $P(A)P(B)$	compare, then conclude in words D3
"mutually exclusive"	$P(A \cap B) = 0$	no overlap; not the same as independent W2

In the exam 1 *command terms*

Given that: divide by the probability of the condition. **Show that ... independent**: compute $P(A)P(B)$, compare with $P(A \cap B)$, conclude. **Exact**: fractions on Paper 1.

VI ON THE GDC: PAPER 2

SL 4.5 · 4.6 · 4.11 · NumWorks · every calculator move of this topic

1 Fractions kept exact

Calculation → 15/56 ÷ 30/56 → EXE

The NumWorks keeps fractions exact; the ↔ key switches to the decimal. **On paper:** the fraction, or the decimal to 3 s.f.

CALCULATION

```
(15/56)/(30/56)
1/2
0.35*0.25+0.65*0.6
0.4775
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ANSWERS TO THE APPLY QUESTIONS

I.1 $\frac{6}{20} = \frac{3}{10}$

I.2 0.58

I.3 15

I.4 2.1

II.1 0.85

II.2 0.2

II.3 5

II.4 $\frac{7}{30}$

II.5 $\frac{1}{8}$

II.6 $1 - 0.49 = 0.51$

II.7 $\frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

III.1 0.3

III.2 $\frac{15}{35} = \frac{3}{7}$

III.3 0.029; 0.517

III.4 no: $P(A \cap B) = 0.15 \neq 0.2$

III.5 0.76

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