The normal distribution

SL AA · Statistics and probability · 4.9 · 4.12

Heights, masses, measurement errors: many continuous quantities pile up symmetrically around their mean, in the shape of a bell. A probability is then an area under that bell. Two questions cover the topic: from a value to a probability, and from a probability back to a value.

I THE NORMAL DISTRIBUTION

SL 4.9 · $X \sim N(\mu, \sigma^2)$, probabilities as areas, symmetry

I.1 A probability is an area

Definition 1 *normal distribution*

$X \sim N(\mu, \sigma^2)$: a continuous random variable whose distribution is the bell-shaped curve symmetric about the mean μ, with standard deviation σ. $P(a < X < b)$ is the area under the curve from a to b; the total area is 1.

Example 1

$X \sim N(40, 6^2)$. Find $P(X < 45)$ and $P(X > 30)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Write the probability statement	$P(X < 45)$
Step 2 Normal cdf on the GDC: $\mu = 40, \sigma = 6$	0.798
Step 3 The same for the other	$P(X > 30) = 0.952$

APPLY

1 $N(100, 15^2)$: $P(X > 120)$.

2 $N(100, 15^2)$: $P(X < 88)$.

I.2 Between two values

Watch out 1

For a continuous variable, $P(X < a) = P(X \leq a)$ and $P(X = a) = 0$. Write the statement, for example $P(X > 185)$, before the number.

Example 2

$X \sim N(40, 6^2)$. Find $P(35 < X < 50)$, and the expected number of values between 35 and 50 in a sample of 200.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Lower and upper bounds in the normal cdf	$P(35 < X < 50) = 0.750$
Step 2 Expected number: $n\times$ probability	$200 \times 0.74988\ldots = 150$

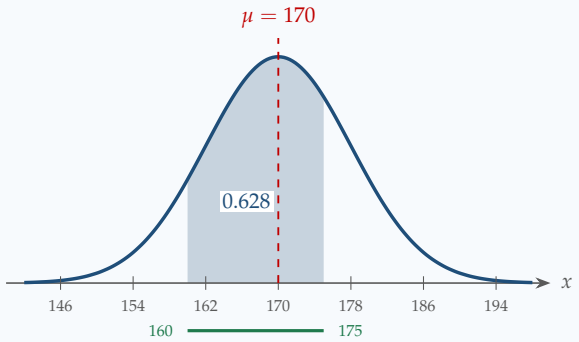


Figure 1 — Exam-style question 1: $N(170, 8^2)$, the area between 160 and 175 is 0.628.

APPLY

3 $N(100, 15^2)$: $P(90 < X < 110)$.

4 $N(100, 15^2)$, 300 people: the expected number above 120.

I.3 Symmetry, without a calculator

Property 1 *symmetry and the spread*

$P(X < \mu) = P(X > \mu) = 0.5$ and $P(X < \mu - k) = P(X > \mu + k)$. About 68 % of the values lie within σ of μ, 95 % within 2σ, 99.7 % within 3σ.

Example 3

X is normal with mean 25 and $P(X > 30) = 0.2$. Find $P(X < 20)$, $P(20 < X < 30)$ and $P(X < 30)$.

WHAT YOU DO	WHAT YOU WRITE
Step 1 20 and 30 are 5 either side of 25	$P(X < 20) = P(X > 30) = 0.2$
Step 2 Total area 1	$P(20 < X < 30) = 1 - 0.2 - 0.2 = 0.6$
Step 3 The complement	$P(X < 30) = 0.8$

APPLY

5 Mean 12, $P(X < 9) = 0.35$: find $P(X > 15)$.

6 Mean 12, $P(X < 9) = 0.35$: find $P(9 < X < 12)$.

II INVERSE NORMAL AND STANDARDISATION

SL 4.9 · 4.12 · inverse normal, $z = \frac{x-\mu}{\sigma}$, unknown μ or σ

II.1 The inverse normal

Property 2 *inverse normal*

Given a probability p, the GDC finds k with $P(X < k) = p$: the **inverse normal**, always with the area to the **left** of k.

Watch out 2

"The top 10 %" means $P(X > k) = 0.1$: the area to the left is 0.9.

Example 4

$X \sim N(40, 6^2)$. Find k with $P(X < k) = 0.85$, and the value exceeded by the top 10 %.

WHAT YOU DO	WHAT YOU WRITE
Step 1 Area to the left 0.85	$k = 46.2$
Step 2 Top 10 %: area to the left 0.9	$P(X < m) = 0.9$
Step 3 Inverse normal	$m = 47.7$

APPLY

1 $N(100, 15^2)$: the value exceeded by 25 %.

2 $N(100, 15^2)$: k with $P(X < k) = 0.05$.

II.2 The standardised value z

Definition 2 *standardised value*

$z = \dfrac{x-\mu}{\sigma}$: how many standard deviations x is from the mean. $Z = \dfrac{X-\mu}{\sigma} \sim N(0, 1)$, the **standard normal**, and $P(X < x) = P(Z < z)$.

Example 5

Ana scores 72 in a test where the scores are $N(60, 8^2)$; Ben scores 80 in a test where they are $N(70, 5^2)$. Who did better compared with the others?

WHAT YOU DO	WHAT YOU WRITE
Step 1 Ana's z	$\dfrac{72-60}{8} = 1.5$
Step 2 Ben's z	$\dfrac{80-70}{5} = 2$
Step 3 Compare	Ben: 2 standard deviations above the mean

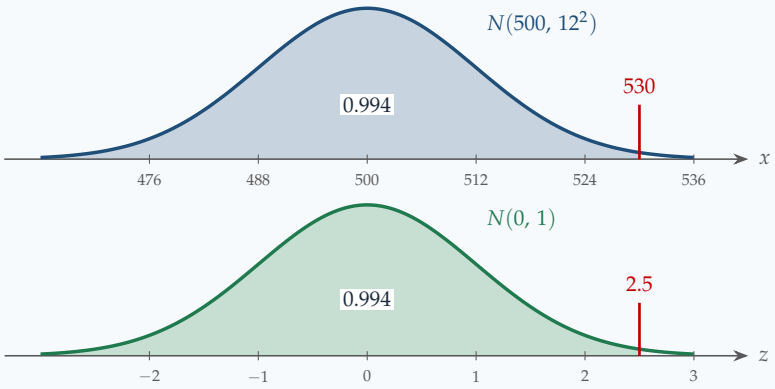


Figure 2 — Exam-style question 3: $x = 530$ in $N(500, 12^2)$ and $z = 2.5$ in $N(0, 1)$, the same area 0.994.

APPLY

3 $N(80, 10^2)$: the z-score of 95.

4 $N(80, 10^2)$: the value with $z = -2$.

II.3 An unknown mean or standard deviation

Property 3 *an unknown mean or standard deviation*

μ or σ unknown, a probability given: find z with the inverse normal on $N(0, 1)$, then solve $z = \dfrac{x - \mu}{\sigma}$.

Example 6

$X \sim N(\mu, 3^2)$ and $P(X < 50) = 0.8$. Find μ.

WHAT YOU DO	WHAT YOU WRITE
Step 1 z from $N(0, 1)$, area to the left 0.8	$z = 0.841621\ldots$
Step 2 Write $z = \dfrac{x - \mu}{\sigma}$	$\dfrac{50 - \mu}{3} = 0.841621\ldots$
Step 3 Solve	$\mu = 47.5$

APPLY

5 $X \sim N(20, \sigma^2)$ and $P(X > 26) = 0.1$: find σ.

6 $X \sim N(\mu, 2^2)$ and $P(X > 15) = 0.3$: find μ.

III EXAM-STYLE QUESTIONS

from the shortest to the longest · every one in steps, with its marks

A Probabilities on the GDC

EXAM-STYLE QUESTION 1 Paper 2 · calculator · [Maximum mark: 6]

The heights of adults in a town are normally distributed with mean 170 cm and standard deviation 8 cm.

(a) Find the probability that an adult chosen at random is shorter than 180 cm. [2]

(b) Find the probability that an adult chosen at random is between 160 cm and 175 cm tall. [2]

(c) In a group of 500 adults, find the expected number who are taller than 185 cm. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write the distribution and the statement	$X \sim N(170,\, 8^2),\ P(X < 180)$	(M1)
Step 2 Normal cdf	$0.894\ \ (0.894350\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Both bounds in the normal cdf	$P(160 < X < 175)$	(M1)
Step 2 Compute	$0.628\ \ (0.628364\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The probability first	$P(X > 185) = 0.0303963\ldots$	(A1)
Step 2 Expected number: $n\times$ probability	$500 \times 0.0303963\ldots = 15.2$	A1

An expected number may be a decimal.

B Symmetry, without a calculator

EXAM-STYLE QUESTION 2 Paper 1 · no calculator · [Maximum mark: 4]

A random variable X is normally distributed with mean 20. It is given that $P(X > 24) = 0.15$.

(a) Write down $P(X < 16)$. [1]

(b) Find $P(16 < X < 24)$. [2]

(c) Find $P(X > 16)$. [1]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Symmetric about μ	0.15 *16 and 24 are symmetric about 20.*	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Total area 1	$1 - 0.15 - 0.15$	(M1)
Step 2 Compute	$= 0.7$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 The complement	$1 - 0.15 = 0.85$	A1

C Inverse normal and z (Paper 2)

EXAM-STYLE QUESTION 3 — Paper 2 · calculator · [Maximum mark: 5]

The masses of bags of flour are normally distributed with mean 500 g and standard deviation 12 g.

(a) Find the mass exceeded by 5 % of the bags. [2]

(b) Find the value of k such that $P(X < k) = 0.2$. [1]

(c) Find the standardised value of a mass of 530 g. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Top 5 %: area to the left 0.95	$P(X < m) = 0.95$ *Area to the left:* $1 - 0.05$.	(M1)
Step 2 Inverse normal	$m = 520$ g $\ (519.738\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Inverse normal, area 0.2	$k = 490$ g $\ (489.900\ldots)$	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 $z = \dfrac{x - \mu}{\sigma}$	$z = \dfrac{530 - 500}{12}$	(M1)
Step 2 Compute	$= 2.5$	A1

EXAM-STYLE QUESTION 4

Paper 2 · calculator · [Maximum mark: 6]

The random variable X is normally distributed with mean 60 and standard deviation σ. It is given that $P(X > 70) = 0.1$.

(a) Find σ. [3]

(b) Write down $P(X < 50)$. [1]

The random variable Y is normally distributed with mean μ and standard deviation 4. It is given that $P(Y < 30) = 0.25$.

(c) Find μ. [2]

WORKING (a)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Standardise	$P\left(Z < \dfrac{70-60}{\sigma}\right) = 0.9$ *Standardise: the inverse normal of X needs σ.*	(M1)
Step 2 z from $N(0, 1)$	$\dfrac{10}{\sigma} = 1.28155\ldots$	(A1)
Step 3 Solve for σ	$\sigma = 7.80$ $(7.80304\ldots)$	A1

WORKING (b)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Symmetric about μ	0.1 *50 and 70 are symmetric about 60.*	A1

WORKING (c)

WHAT YOU DO	WHAT YOU WRITE	
Step 1 Write $z = \dfrac{x-\mu}{\sigma}$	$\dfrac{30-\mu}{4} = -0.674489\ldots$	(M1)
Step 2 Solve for μ	$\mu = 32.7$ $(32.6979\ldots)$	A1

IV WHICH METHOD? READ THE QUESTION

4.12 · the whole topic on one page

THE QUESTION SAYS	YOU KNOW	YOU WRITE
"find the probability that $X < a$"	an area to the left	$P(X < a)$ on the GDC, normal cdf D1
"between a and b"	an area between	$P(a < X < b)$ D1
no GDC, a probability given	symmetry about μ	$P(X < \mu - k) = P(X > \mu + k)$; total 1 P1
"the value exceeded by p"	the inverse normal	area to the left $1 - p$ P2
"standardised value", "z-score"	$z = \frac{x - \mu}{\sigma}$	how many σ from μ D2
μ or σ unknown	z from $N(0, 1)$	inverse normal on Z, then solve for μ or σ P3
"expected number"	$n\times$ probability	the probability first, then multiply

In the exam 1 *command terms*

State the distribution: $X \sim N(\mu, \sigma^2)$ with the numbers. **Find the probability**: the statement, then the value to 3 s.f. **Find the value**: the inverse normal, with the area to the left.

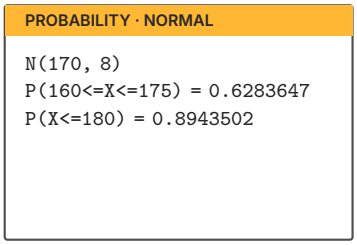
V ON THE GDC: PAPER 2

SL 4.9 · 4.12 · NumWorks · every calculator move of this topic

1 A probability: the normal cdf

Probability → Normal → μ, σ → P(a ≤ X ≤ b)

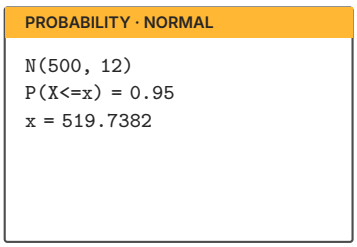
Choose the normal law, enter μ and σ (not σ^2), then the interval. **On paper:** "$P(160 < X < 175) = 0.628$".



2 A value: the inverse normal

Probability → Normal → μ, σ → P(X ≤ x) = p

Enter the area to the left; the GDC returns x. **On paper:** "$P(X < m) = 0.95$, $m = 520$ g".



ANSWERS TO THE APPLY QUESTIONS

I.1 0.0912

I.2 0.212

I.3 0.495

I.4 27.4

I.5 0.35

I.6 0.15

II.1 110

II.2 75.3

II.3 1.5

II.4 60

II.5 $\frac{6}{\sigma} = 1.28155$: 4.68

II.6 14.0

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