Equations of planes

Skills sheet · HL AA · AHL 3.17

KEY FACTS

$\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n},\ ax+by+cz=d$ D1

$\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c},\ \mathbf{n}=\mathbf{b}\times\mathbf{c}$ D2

three points: $\mathbf{n}=\overrightarrow{AB}\times\overrightarrow{AC}$ P1

parallel planes: the same normal P2

to parametric: a point, two directions $\perp\mathbf{n}$ P3

$d=\mathbf{a}\cdot\mathbf{n}$, from a point W1.
Check every point in the equation W2.
The coefficients are the normal D1.

WHAT UNLOCKS

- a point and a normal $\rightarrow$ d in one line.
- three points $\rightarrow$ two vectors, their product, then d.
- "perpendicular to the plane" $\rightarrow$ the normal as a direction.
- "parallel to the plane through P" $\rightarrow$ same (a, b, c), new d.

WORKED EXAMPLE ①

Find the plane through $(1, 2, -1)$ with normal $\begin{pmatrix}3\\-1\\2\end{pmatrix}$. no GDC

$3-2-2=-1$ M1

$3x-y+2z=-1$ A1

Method

Point and normal

① The coefficients from $\mathbf{n}$.
② d from the point.
③ Write $ax+by+cz=d$.

WORKED EXAMPLE ②

Find the Cartesian equation of $\mathbf{r}=\begin{pmatrix}1\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}+\mu\begin{pmatrix}0\\1\\1\end{pmatrix}$. no GDC

$\mathbf{n}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ M1

$x-y+z=1$ A1

Method

Parametric to Cartesian

① $\mathbf{n}=\mathbf{b}\times\mathbf{c}$.
② $d=\mathbf{a}\cdot\mathbf{n}$.
③ Check one more point.

FLUENCY

●○○ **1.** A normal to $4x-y+7z=2$ no GDC

●○○ **2.** Plane through $(2, 3, -1)$ with normal $\begin{pmatrix}1\\1\\1\end{pmatrix}$ no GDC

●○○ **3.** Is $(1, 2, 3)$ in $2x+y-z=1$? no GDC

●○○ **4.** Plane parallel to $x-2y+2z=5$ through the origin no GDC

●○○ **5.** Line through $(1, 1, 1)$ perpendicular to $2x+3y-z=4$ no GDC

●○○ **6.** A normal to the plane $z=3$ no GDC

REASONING

●●● **Question 1.** The points $A(1, 0, 0)$, $B(0, 2, 0)$ and $C(0, 0, 3)$ are given. [no GDC]

(a) Find the Cartesian equation of the plane ABC. [4]

●●● **Question 2.** The plane Π has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. [no GDC]

(a) Find the Cartesian equation of Π. [4]

●●● **Question 3.** [no GDC]

(a) Find the value of k for which the planes $2x + ky - z = 3$ and $4x - 6y - 2z = 1$ are parallel. [2]

PROBLEMS

●●● **Question 4.** The point $P(5, 1, 4)$ and the plane $x + y + 2z = 4$ are given. [no GDC]

(a) Find the coordinates of the foot of the perpendicular from P to the plane. [3]

(b) Hence find the distance from P to the plane. [2]

●●● **Question 5.** The plane Π contains $A(0.5, 1, -2)$, $B(2, -1.5, 1)$ and $C(-1, 3, 0.5)$. [with GDC]

(a) Find a vector normal to Π. [2]

(b) Find the coordinates of the point where Π meets the z-axis. [2]

ANSWERS

1 $\begin{pmatrix} 4 \\ -1 \\ 7 \end{pmatrix}$

2 $x + y + z = 4$

3 yes: $2 + 2 - 3 = 1$

4 $x - 2y + 2z = 0$

5 $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

6 $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Q1 $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$, $d = 6$: $6x + 3y + 2z = 6$

Q2 $\mathbf{n} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$, $d = -3$: $2x - y - z = 3$

Q3 $\frac{2}{4} = \frac{k}{-6}$: $k = -3$

Q4 $\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$: $14 + 6t = 4$, $t = -\frac{5}{3}$: $\left(\frac{10}{3}, -\frac{2}{3}, \frac{2}{3}\right)$; distance $\frac{5}{3}\sqrt{6} = \frac{5\sqrt{6}}{3}$

Q5 $\begin{pmatrix} -12.25 \\ -8.25 \\ -0.75 \end{pmatrix}$, or $\begin{pmatrix} 49 \\ 33 \\ 3 \end{pmatrix}$; $49x + 33y + 3z = 51.5$: $(0, 0, 17.2)$ $(z = \frac{103}{6})$

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