The vector product

Skills sheet · HL AA · AHL 3.16

KEY FACTS

$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$ D1

$\mathbf{w} \times \mathbf{v} = -\mathbf{v} \times \mathbf{w}$; parallel $\iff \mathbf{0}$ P1

$|\mathbf{v} \times \mathbf{w}| = |\mathbf{v}||\mathbf{w}| \sin\theta$: parallelogram P2

triangle $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$ P2

distance to a line: area over base P3

Middle component: $v_3w_1 - v_1w_3$ W1.

Check: $\cdot\,\mathbf{v} = 0$ and $\cdot\,\mathbf{w} = 0$ D1.

Both vectors from one vertex W2.

WHAT UNLOCKS

- "perpendicular to both" $\rightarrow$ the vector product.
- "a unit vector perpendicular to both" $\rightarrow$ divide by its magnitude.
- "the area of the triangle" $\rightarrow$ half the magnitude.
- "the distance from C to the line AB" $\rightarrow \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{|\overrightarrow{AB}|}$.

WORKED EXAMPLE ①

Find $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$. no GDC

$\begin{pmatrix} 0\times1 - 2\times3 \\ 2\times0 - 1\times1 \\ 1\times3 - 0\times0 \end{pmatrix}$ M1

$= \begin{pmatrix} -6 \\ -1 \\ 3 \end{pmatrix}$ A1

Method

A vector product

① Write the three cross terms.
② Mind the middle sign.
③ Check one scalar product is 0.

WORKED EXAMPLE ②

Find the area of the parallelogram with sides $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$. no GDC

$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ M1

area $= \sqrt{3}$ A1

Method

An area

① The vector product.
② Its magnitude.
③ Half of it for a triangle.

FLUENCY

●○○ **1.** $\mathbf{i} \times \mathbf{j}$ no GDC

●○○ **2.** $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ no GDC

●○○ **3.** $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ no GDC

●○○ **4.** $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ no GDC

●○○ **5.** $\mathbf{b} \times \mathbf{a}$ when $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ no GDC

●○○ **6.** Area of the triangle with $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$ no GDC

REASONING

●●● **Question 1.** [no GDC]

(a) Find a unit vector perpendicular to both $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$. [3]

●●● **Question 2.** The points $A(1, 0, 0)$, $B(0, 2, 0)$ and $C(0, 0, 3)$ are given. [no GDC]

(a) Find the area of the triangle ABC. [3]

●●● **Question 3.** Let $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. [no GDC]

(a) Find $\mathbf{a} \times \mathbf{b}$ and verify that it is perpendicular to $\mathbf{a}$. [3]

PROBLEMS

●●● **Question 4.** The line L passes through the origin with direction $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$. [no GDC]

(a) Find the exact distance from $C(2, 3, 1)$ to L. [4]

●●● **Question 5.** $ABCD$ is a parallelogram with $A(1.2, 0, -0.5)$, $B(3, 2.4, 1)$ and $D(-1, 1.5, 2)$. [with GDC]

(a) Find its area. [3]

ANSWERS

1 $\mathbf{k}$

2 $\begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$

3 $\mathbf{0}$

4 $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

5 $\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$

6 $\frac{1}{2} \times 12 = 6$

Q1 $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$, magnitude $\sqrt{6}$: $\frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

Q2 $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$, magnitude 7: area $\frac{7}{2}$

Q3 $\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$; $\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = -8 - 1 + 9 = 0$

Q4 $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ -1 \end{pmatrix}$, magnitude $\sqrt{26}$; distance $\frac{\sqrt{26}}{3}$

Q5 $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{pmatrix} 3.75 \\ -7.8 \\ 7.98 \end{pmatrix}$: area 11.8 (11.7721…)

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