Vectors and the scalar product

Skills sheet · HL AA · AHL 3.12 · 3.13

KEY FACTS

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ D1
$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$, unit vector $\frac{1}{|\mathbf{v}|}\mathbf{v}$ D2
$\mathbf{w} = k\mathbf{v}$: parallel; $\mathbf{d} = \mathbf{a} + \mathbf{c} - \mathbf{b}$ P1
$\mathbf{v} \cdot \mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3 = |\mathbf{v}||\mathbf{w}| \cos\theta$ D3
perpendicular $\iff$ $\mathbf{v} \cdot \mathbf{w} = 0$ P2

End minus start, never $\mathbf{a} - \mathbf{b}$ W1.
Angle at B: the vectors leaving B W2.
$\mathbf{v} \cdot \mathbf{w} < 0$*: an obtuse angle* P2.

WHAT UNLOCKS

- "the distance AB" $\rightarrow$ $\left|\overrightarrow{AB}\right|$, one square root.
- "collinear" $\rightarrow$ $\overrightarrow{AC} = k\overrightarrow{AB}$ and a shared point.
- "perpendicular" with an unknown $\rightarrow$ the scalar product equals 0.
- "the angle between" $\rightarrow$ scalar product over the two magnitudes, then arccos.

WORKED EXAMPLE ①

Find the distance between $A(1, 2, -1)$ and $B(3, -4, 2)$. no GDC

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix}$ M1
$AB = \sqrt{4 + 36 + 9} = 7$ A1

Method
A distance
① $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.
② Square, add, root.
③ A whole number when it can be.

WORKED EXAMPLE ②

Find the angle between $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$. no GDC

$\cos\theta = \frac{0+1+0}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2}$ M1
$\theta = 60°$ A1

Method
An angle
① The scalar product.
② The two magnitudes.
③ arccos of the quotient.

FLUENCY

●○○ **1.** $\overrightarrow{AB}$ for $A(3, -1, 2)$, $B(1, 4, 2)$ no GDC

●○○ **2.** $\left|\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}\right|$ no GDC

●○○ **3.** Unit vector along $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ no GDC

●○○ **4.** $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 5 \end{pmatrix}$ no GDC

●○○ **5.** t with $\begin{pmatrix} t \\ 2 \\ -1 \end{pmatrix} \perp \begin{pmatrix} 3 \\ t \\ 4 \end{pmatrix}$ no GDC

●○○ **6.** Are $\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$ parallel? no GDC

REASONING

●●● **Question 1.** The points $A(1, 0, 2)$, $B(2, 3, 1)$ and $C(4, 9, -1)$ are given. [no GDC]

(a) Show that A, B and C are collinear. [3]

●●● **Question 2.** Let $\mathbf{v} = \begin{pmatrix} k \\ 3 \\ 2k \end{pmatrix}$. [no GDC]

(a) Find the values of k for which $|\mathbf{v}| = 3\sqrt{6}$. [3]

●●● **Question 3.** The points $A(2, 1, 2)$, $B(0, 1, 2)$ and $C(1, 2, 2)$ are given. [no GDC]

(a) Find the exact size of the angle $A\hat{B}C$. [4]

PROBLEMS

●●● **Question 4.** The points $A(1, 1, 1)$, $B(3, 2, -1)$ and $C(4, k, 3)$ form a triangle with a right angle at A. [no GDC]

(a) Find the value of k. [3]

(b) Find the exact area of the triangle. [2]

●●● **Question 5.** A cuboid has vertices $O(0, 0, 0)$, $A(2, 0, 0)$, $P(2, 3, 6)$ and $Q(0, 3, 6)$. [with GDC]

(a) Find the acute angle between the diagonals OP and AQ. [4]

ANSWERS

1 $\begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix}$

2 7

3 $\frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

4 $6 - 4 - 5 = -3$

5 $3t + 2t - 4 = 0,\ t = \frac{4}{5}$

6 yes: $\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = -2\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$

Q1 $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 3 \\ 9 \\ -3 \end{pmatrix} = 3\overrightarrow{AB}$: parallel, with the common point A, so collinear

Q2 $k^2 + 9 + 4k^2 = 54,\ k^2 = 9,\ k = \pm 3$

Q3 $\overrightarrow{BA} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$: $\cos\theta = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}},\ \theta = \frac{\pi}{4}\ (45°)$

Q4 $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 3 \\ k-1 \\ 2 \end{pmatrix}$: $6 + k - 1 - 4 = 0,\ k = -1$; area $\frac{1}{2} \times 3 \times \sqrt{17} = \frac{3\sqrt{17}}{2}$

Q5 $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}, \overrightarrow{AQ} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$: $\cos\theta = \frac{41}{49},\ \theta = 33.2°$

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