

Marked example

Paper 1 style · No calculator

14

14 marks SL 5.6 · 5.7 · 5.8

●●● Typical to demanding

Let $f(x) = (x^2 - 3)e^x$, for $x \in \mathbb{R}$.

- (a) Show that $f'(x) = (x + 3)(x - 1)e^x$. [3]
- (b) Find the x -coordinates of the stationary points of the graph of f . [2]
- (c) Show that the graph of f has a local maximum at $x = -3$, and find the exact value of this local maximum. [3]

It is given that $f''(x) = (x^2 + 4x - 1)e^x$.

- (d) Find the exact x -coordinates of the points of inflexion of the graph of f . Justify your answer. [3]

It is also given that $f(x) \rightarrow 0$ as $x \rightarrow -\infty$.

- (e) Find the set of values of k for which the equation $f(x) = k$ has exactly three solutions. [3]

MARKSCHEME (a)

attempt to use the product rule

M1

$$f'(x) = 2xe^x + (x^2 - 3)e^x$$

A1

$$= (x^2 + 2x - 3)e^x$$

A1

$$= (x + 3)(x - 1)e^x$$

AG

Total 3 marks

Note The intermediate form $(x^2 + 2x - 3)e^x$, or equivalent, must be seen for the second A1.

MARKSCHEME (b)

$$f'(x) = 0 \text{ and } e^x > 0 \text{ (or } e^x \neq 0), \text{ so } (x + 3)(x - 1) = 0$$

(M1)

$$x = -3, x = 1$$

A1

Total 2 marks

MARKSCHEME (c)

EITHER

sign of f' : positive for $x < -3$, negative for $-3 < x < 1$ (sign diagram or words)

M1

f' changes from positive to negative at $x = -3$, so local maximum

R1

OR

$$f''(-3) = (9 - 12 - 1)e^{-3} = -4e^{-3}$$

M1

$f''(-3) < 0$, so local maximum

R1

THEN

$$\text{local maximum value } f(-3) = 6e^{-3}$$

A1

Total 3 marks

Note Accept $\frac{6}{e^3}$. A decimal value only earns A0.

MARKSCHEME (d)

$$f''(x) = 0 \Rightarrow x^2 + 4x - 1 = 0 \text{ (since } e^x > 0\text{)}$$

M1

$$x = -2 \pm \sqrt{5}$$

A1

$x^2 + 4x - 1$ has two distinct roots, so f'' changes sign (concavity changes) at each of them

R1**Total 3 marks**

Note Accept a correct sign diagram of f'' for R1. Do not award R1 for " $f''(x) = 0$ " alone.

MARKSCHEME (e)

f increases from 0 to $6e^{-3}$ for $x < -3$, decreases to $-2e$ for $-3 < x < 1$, then increases without bound (words or a sketch)

R1

$$k > 0$$

A1

$$k < 6e^{-3}$$

A1**Total 3 marks**

Note Both inequalities must be strict. Accept $0 < k < \frac{6}{e^3}$ and interval notation such as $]0, 6e^{-3}[$. A candidate who includes $k = 0$ or negative values of k has not used the fact that f increases from 0 for $x < -3$: award R0A0A1 at most. FT from their local maximum in (c).