

Marked example

Paper 2 style · Graphic display calculator

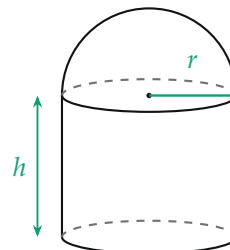
25

15 marks SL 5.8

Typical to demanding

A sensor housing is made from thin sheet metal. It consists of a cylinder of radius r cm and height h cm, closed at the base, with a hemispherical cap of radius r cm on top, as shown in the diagram. There is no metal between the cylinder and the cap.

The volume enclosed by the housing is 500 cm^3 .



(a) Show that $h = \frac{500}{\pi r^2} - \frac{2r}{3}$. [2]

(b) Show that the total external surface area, $S \text{ cm}^2$, of the housing is given by $S = \frac{5}{3}\pi r^2 + \frac{1000}{r}$. [3]

(c) Find $\frac{dS}{dr}$. [2]

(d) Find the value of r for which S is a minimum. Justify that it is a minimum. [3]

(e) Show that, for this value of r , the height of the cylinder is equal to its radius. [2]

The housing must fit inside a tube, so that its radius can be at most 4 cm.

(f) Find the least possible surface area of a housing that fits inside the tube. Justify your answer. [3]

MARKSCHEME (a)

$$\pi r^2 h + \frac{2}{3}\pi r^3 = 500$$

$$h = \frac{500}{\pi r^2} - \frac{2r}{3}$$

M1A1

AG

Total 2 marks

Note M1 for adding the volumes of a cylinder and of a hemisphere; A1 for the correct equation. The AG line must follow from correct working.

MARKSCHEME (b)

$$S = \pi r^2 + 2\pi r h + 2\pi r^2 \text{ (base, curved surface, hemisphere)}$$

$$2\pi r \left(\frac{500}{\pi r^2} - \frac{2r}{3} \right) = \frac{1000}{r} - \frac{4}{3}\pi r^2$$

$$S = \frac{5}{3}\pi r^2 + \frac{1000}{r}$$

M1A1

A1

AG

Total 3 marks

Note M1 for a sum of three areas; A1 for all three correct, including $2\pi r^2$ for the hemisphere.

MARKSCHEME (c)

$$\frac{dS}{dr} = \frac{10}{3}\pi r - \frac{1000}{r^2}$$

A1A1

Total 2 marks

MARKSCHEME (d)

$$\frac{10}{3}\pi r - \frac{1000}{r^2} = 0, \text{ or the minimum of } S \text{ on the GDC}$$

(M1)

$$r = 4.57 \text{ (4.57078...)}$$

A1

EITHER

$$\frac{d^2S}{dr^2} = \frac{10}{3}\pi + \frac{2000}{r^3} > 0 \text{ for } r > 0, \text{ so minimum}$$

R1

OR

$$\frac{dS}{dr} \text{ changes from negative to positive at } r = 4.57, \text{ so minimum}$$

R1**Total 3 marks****Note** Accept a GDC graph of S with a clearly identified minimum point for R1.**MARKSCHEME (e)**

$$\text{from (d), } \frac{10}{3}\pi r^3 = 1000, \text{ so } \pi r^3 = 300$$

M1

$$h = \frac{500}{\pi r^2} - \frac{2r}{3} = \frac{500r}{\pi r^3} - \frac{2r}{3} = \frac{5r}{3} - \frac{2r}{3}$$

A1

$$h = r$$

AG**Total 2 marks****Note** A numerical check only (e.g. substituting $r = 4.57$ and obtaining $h \approx 4.57$) earns M0A0: the question asks for an exact argument.**MARKSCHEME (f)**

$$\frac{dS}{dr} < 0 \text{ for } 0 < r < 4.57, \text{ so } S \text{ is decreasing for } 0 < r \leq 4$$

R1

$$\text{so the least value is at } r = 4: S = \frac{5}{3}\pi(16) + \frac{1000}{4}$$

M1

$$S = 334 \text{ cm}^2 \text{ (333.776...)}$$

A1**Total 3 marks****Note** Award R1 for any valid argument that the least value on $0 < r \leq 4$ occurs at $r = 4$, including a GDC graph of S restricted to this interval. An answer of 328 (the unconstrained minimum) earns R0M0A0.**ON THE CALCULATOR****TI-Nspire CX II**

(d) Graph: $f1(x) = \frac{5}{3}\pi x^2 + \frac{1000}{x}$; menu → 4 Window/Zoom → 1 Window Settings: x from 0 to 10, y from 0 to 600. menu → 6 Analyze Graph → 2 Minimum, bounds 1 and 9 → (4.57078, 328.171).

NumWorks

(d) Functions: $f(x) = \frac{5}{3}\pi x^2 + \frac{1000}{x}$. Graph → Axes → Values of X: 0 to 10; Values of Y: 0 to 600. Graph → Calculate → Find → Minimum → $x = 4.570781$, $f(x) = 328.1710$.