Random variables, binomial and normal

tabulate → count → translate → measure → invert → choose

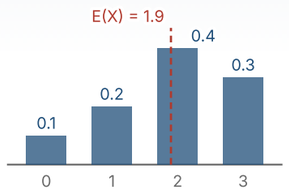
I Discrete random variables

AA 4.7

$\Sigma P(X = x) = 1 \quad E(X) = \Sigma x\, P(X = x)$

*fair game: expected **gain** = 0*

x	0	1	2	3
$P(X = x)$	0.1	k	2k	0.3



Missing value $0.4 + 3k = 1$, $k = 0.2$ · **expectation** $E(X) = 0.2 + 0.8 + 0.9 = 1.9$.

Is the game fair? Pay $2; win $5 with probability 0.3, $1 with probability 0.2: expected gain $5(0.3) + 1(0.2) - 2 = -0.3 \neq 0$: **not fair**, the player loses 30 cents a game on average.

Expected number of times in n trials: $n \times P$ — it may be a decimal.

Watch out E(X) need not be a possible value of X · in a game, subtract the stake from **every** outcome, or once at the end — not both.

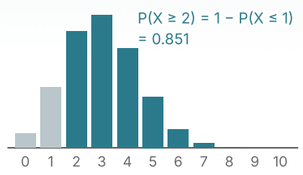
II The binomial distribution

AA 4.8

$X \sim B(n, p)$: $P(X = r) = {}^nC_r p^r (1-p)^{n-r}$

$E(X) = np \cdot Var(X) = np(1-p)$

Conditions: fixed number n of trials, two outcomes, constant p, trials independent — state them **in context**.



Exactly $X \sim B(10, 0.3)$: $P(X = 2) = 45(0.3)^2(0.7)^8 \approx 0.233$ — GDC **binompdf**.

At least $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.149 = 0.851$ — GDC **binomcdf** gives P(X ≤ k).

Mean and variance $E(X) = 3$, $Var(X) = 2.1$, standard deviation $\sqrt{2.1} \approx 1.45$.

Watch out without replacement from a small group, p changes: **not** binomial · the variance is $np(1-p)$, not its square root.

III Binomial: words to inequalities

AA 4.8 · 4.6

the question says	you calculate
"at most 3"	$P(X \leq 3)$
"fewer than 3"	$P(X \leq 2)$
"more than 3"	$1 - P(X \leq 3)$
"at least 3"	$1 - P(X \leq 2)$
"between 2 and 5 inclusive"	$P(X \leq 5) - P(X \leq 1)$

More than 3 $X \sim B(15, 0.2)$: $1 - P(X \leq 3) \approx 0.352$.

Least number of trials P(at least one success) ≥ 0.95 with $p = 0.1$: $1 - 0.9^n \geq 0.95$, $n \geq \ln 0.05 / \ln 0.9 \approx 28.4$, so **n = 29**.

Conditional $P(X = 2 \mid X \geq 1) = \frac{P(X = 2)}{1 - P(X = 0)} \approx 0.240$.

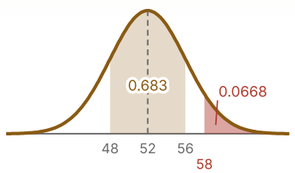
Watch out on a discrete variable, "more than 3" starts at 4 · the inequality on n flips when you divide by ln 0.9 < 0.

IV The normal distribution

AA 4.9

$X \sim N(\mu, \sigma^2)$

symmetric about μ · total area 1 · $P(X = a) = 0$



About 68 % within one σ of μ, 95 % within two, 99.7 % within three.

Probability $X \sim N(52, 4^2)$: $P(X > 58) \approx 0.0668$, $P(48 < X < 56) \approx 0.683$ — GDC **normalcdf**(lower, upper, μ, σ).

Expected number out of 500 items, about $500 \times 0.0668 \approx 33$ are above 58.

Sketch first: the bell, μ in the middle, the region shaded — it checks the order of magnitude of the answer.

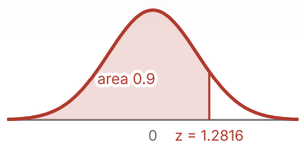
Watch out the GDC wants σ, not σ² · $P(X \leq a) = P(X < a)$ for a normal variable, not for a binomial one.

V Inverse normal and standardisation

AA 4.9 · 4.12

$z = \frac{x - \mu}{\sigma}, \quad Z \sim N(0, 1)$

z = number of standard deviations from the mean



Find k $P(X < k) = 0.9$, $X \sim N(52, 4^2)$: **invNorm**(0.9, 52, 4), $k \approx 57.13$.

Unknown σ μ = 100, $P(X > 110) = 0.2$: $P(Z < 10/\sigma) = 0.8$, $10/\sigma = 0.8416$, $\sigma \approx 11.88$.

Unknown μ σ = 5, $P(X < 40) = 0.15$: $(40 - \mu)/5 = -1.0364$, $\mu \approx 45.18$.

Watch out invNorm takes the area to the **left**: P(X > k) = 0.2 → use 0.8 · an unknown μ or σ **always** goes through z.

VI Which distribution? Mixed questions

AA 4.6 · 4.8 · 4.9

a measurement: mass, time, height	normal
a count of successes in n trials	binomial
a table of values and probabilities	discrete random variable
"given that"	conditional: divide by the condition

Normal, then binomial jars have mass $X \sim N(52, 4^2)$; a jar is "heavy" if $X > 58$, $p \approx 0.0668$. In a box of 12, Y = number of heavy jars, $Y \sim B(12, 0.0668)$: $P(Y \geq 2) \approx 0.189$.

Conditional normal $P(X > 58 \mid X > 52) = \frac{P(X > 58)}{P(X > 52)} = 0.0668/0.5 \approx 0.134$.

Watch out keep the full value of p from the first part in the GDC: rounding to 0.07 changes the third figure of the answer.

Command terms

Write down E(X) = np straight away · **Show that** k = ..., with the sum equal to 1 · **Justify** binomial: the conditions in context · **Find the least n** an integer, with the inequality · **Comment** fair or not, in a sentence.

Paper 1 · Paper 2

P1, no GDC: tables with k, E(X), fair games, symmetry of the normal curve, exact binomial with small n.
P2, GDC: binompdf, binomcdf, normalcdf, invNorm, unknown μ or σ, normal then binomial.

Booklet or memory?

booklet E(X) = Σx P(X = x) · binomial P(X = r), E, Var · z = (x − μ)/σ
by heart the words → inequalities table · conditions for binomial · invNorm uses the left area · sketch the bell

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