Functions

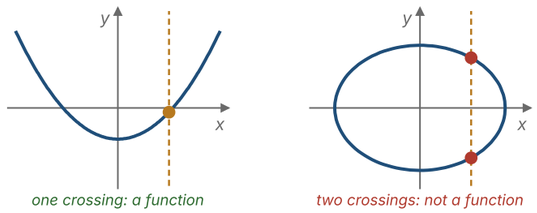
notation → domain & range → key features → compose → invert → transform

I Functions & notation

AA 2.2

Relation = any set of pairs (x, y) · **function** = every input has exactly one output. **One-to-one**: different inputs, different outputs · **many-to-one**: two inputs may share an output.

The vertical line test



one crossing: a function

two crossings: not a function

Function notation

$f(x) = 3x - 5$ means $f(2) = 3(2) - 5 = 1$

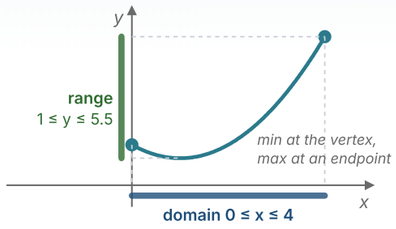
*whatever is in the bracket replaces **every** x — a number, a letter, an expression*

Watch out $f(x + 1)$ is **not** $f(x) + 1$: the first changes the input, the second the answer.

Solving $f(x) = 10$ and finding $f^{-1}(10)$ are the **same task**.

II Domain & range

AA 2.2



Domain: the x allowed — along the x-axis. **Range**: the y produced — along the y-axis.

Same set, three spellings

$-1 < x \leq 5$	$]-1, 5]$	$(-1, 5]$
$x \geq 9$	$[9, \infty[$	$[9, \infty)$

∞ is never reached: always an **open** bracket.

Largest possible domain — only two bans

no division by zero → **solve** denominator $\neq 0$ · no square root of a negative → **solve** inside ≥ 0

*a root **in a denominator**: strictly* > 0 — $\frac{2x+1}{\sqrt{(6+x-x^2)}}$ *needs* $-2 < x < 3$

Watch out restricting the domain changes the range — read the domain **before** the range; the extreme value may sit at an **endpoint**, not at the vertex.

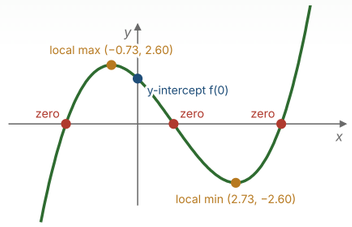
GDC range questions are Paper 2: graph on the given domain, read lowest and highest values.

III Graphs & key features

AA 2.3

Draw ≠ sketch

Draw: accurate, to scale, ruler for lines, points plotted. **Sketch**: general shape with the relevant features labelled.



What the examiner wants labelled

- **zeros** (= roots, x-intercepts)
- **y-intercept** $f(0)$
- **max / min**, vertex, axis of symmetry
- vertical / horizontal **asymptotes**
- the **endpoints** of a given domain

GDC zeros, max/min and intersections are **read**, not calculated — 3 s.f. Solving $f(x) = g(x)$ = intersecting the two graphs.

In a **model**, the context decides the domain: time and height ≥ 0, from launch to landing.

Watch out no axis labels, no scale, no named point: a sketch worth nothing.

Every session

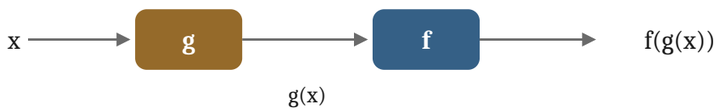
- "Find the range of f for $a \leq x \leq b$" → graph on that domain only; compare the **endpoints** with the turning points.
- "How many solutions has $f(x) = k$?" → the horizontal line $y = k$; count the crossings, mind the turning points.

IV Composite functions

AA 2.5

$(f \circ g)(x) = f(g(x))$

*read "f of g of x" — the function closest to x acts **first***



g acts first: the function written next to x

Inside out

- **A value**: $f(g(3))$ → find $g(3)$ first, then apply f. Works from a **table** or a **graph**.
- **An expression**: substitute the whole of $g(x)$ into f, then expand.
- **Backwards**: $(f \circ g)(x) = 1$ → which input gives $f = 1$? then which x gives g = that?

Watch out $f \circ g \neq g \circ f$ in general — read which one is asked, twice.

Domain of a composite

$g \circ f$: the x in the domain of f **whose image** is in the domain of g. Start from the restriction of the **inside**.

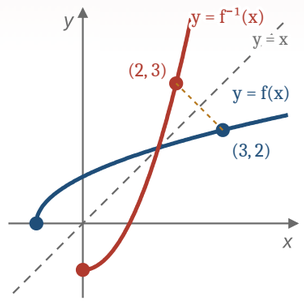
A formula can be **decomposed**: "subtract 100, then × 2.20" is a composite.

V Inverse functions

AA 2.2 · 2.5

$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

Exists only if f is **one-to-one** — otherwise **restrict the domain** (from the vertex on).



Find f^{-1}

1. write $y = f(x)$
2. **swap** x and y
3. make y the subject
4. state the **domain** of f^{-1}

reflection in $y = x$: $(a, b) \to (b, a)$ · domain of f^{-1} = range of f · range of f^{-1} = domain of f

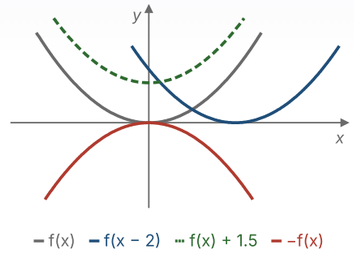
Watch out $f^{-1}(x)$ is **not** $\frac{1}{f(x)}$ · $f^{-1}(a) = b \Leftrightarrow f(b) = a$: often no inverse to compute.

Self-inverse : $f(f(x)) = x$; the graph is symmetric in $y = x$.

VI Transformations

AA 2.11

$f(x) + b$	translation b up
$f(x - a)$	translation a **right**
$p\,f(x)$	vertical stretch, s.f. p
$f(qx)$	horizontal stretch, s.f. $\frac{1}{q}$
$-f(x)$	reflection in the x-axis
$f(-x)$	reflection in the y-axis



A sequence

Inside the bracket first (horizontal), then outside (vertical): stretch, then translate. A point $(3, 4)$ under $3f(x - 2) + 1$ → $(5, 13)$.

Watch out inside the bracket everything works **backwards**: $x - 2$ is 2 to the right, $f(2x)$ is a factor $\frac{1}{2}$ · never write "move": **translation**, **stretch**, **reflection** + direction + factor.

Command terms

Write down / State the answer alone, no working credited · **Find** the working carries the M marks · **Sketch** shape + labelled features · **Hence** use what you just found · **Hence or otherwise** free route.

Paper 1 · Paper 2

P1, no GDC: largest domain, composites, f^{-1} by algebra (numbers chosen to stay exact), transformations described in words.
P2, GDC: range on a domain, zeros, max/min, intersections, models in context. 3 s.f.

Booklet or memory?

by heart nothing on functions is in the booklet: the two domain bans · swap x and y · inside-out order · the six transformations
GDC only zeros · max/min · intersections · range on a domain

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