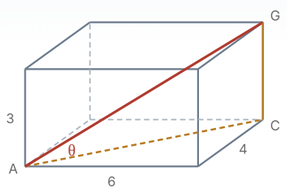
Geometry and trigonometry

solids → triangles → apply → radians → circle → waves

I 3D solids

AA 3.1

distance
$\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2)}$
volumes and surface areas of prisms, pyramids, cones, spheres: in the booklet



Angle with a plane cuboid 6 × 4 × 3: $AC = \sqrt{52}$, $AG = \sqrt{61} \approx 7.81$; $\tan\theta = 3/\sqrt{52}$, $\theta \approx 22.6°$.

1. find the **right-angled triangle** that contains the angle
2. drop the perpendicular to the plane: the projection
3. Pythagoras for the missing side, then SOH CAH TOA

Cone $r = 5$, $h = 12$: slant $l = 13$, $V = \frac{1}{3}\pi r^2 h = 100\pi$, curved area $\pi r l = 65\pi$. **Two points** (1, −2, 3) and (4, 2, 15): distance 13, midpoint (2.5, 0, 9).

Watch out the angle with the base is at A in triangle ACG, not in a face · use the **slant** height for curved area, the vertical height for volume.

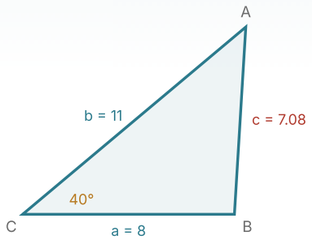
II Sine rule, cosine rule, area

AA 3.2

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
a pair (side, opposite angle) is known

$c^2 = a^2 + b^2 - 2ab\cos C$
two sides and the angle between · or three sides

area $= \frac{1}{2}ab\sin C$



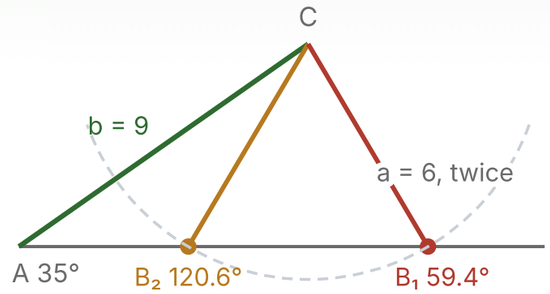
Two sides and the angle between $a = 8$, $b = 11$, $C = 40°$: $c^2 = 64 + 121 - 176\cos 40°$, $c \approx 7.08$; area $= \frac{1}{2}(8)(11)\sin 40° \approx 28.3$.

Then an angle the sine rule gives the angle opposite the **shorter** side: $A \approx 46.5°$, then $B = 180° - 40° - A \approx 93.5°$.

Watch out calculator in **degrees** · asin never returns an obtuse angle: find the largest angle last, by subtraction, or with the cosine rule.

III Ambiguous case, elevation, bearings

AA 3.2 · 3.3



Ambiguous case $A = 35°$, $a = 6$, $b = 9$: $\sin B = 9\sin 35°/6 \approx 0.86$, $B \approx 59.4°$ **or** $B \approx 120.6°$ — both possible, since $35° + 120.6° < 180°$.

Elevation from 40 m away, the top of a mast is at 32° of elevation: $h = 40\tan 32° \approx 25.0$ m.

Bearings: measured from **north**, clockwise, three figures (060°) · the back bearing differs by 180° · draw a north line at **every** point.

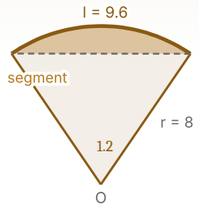
Watch out the ambiguous case appears when the known angle is **opposite the shorter** known side · depression from the top = elevation from the bottom (alternate angles).

IV Radians, arcs and sectors

AA 3.4

π rad $= 180°$
$150° = 5\pi/6 \cdot 1$ rad $\approx 57.3°$

arc $l = r\theta$ sector $A = \frac{1}{2}r^2\theta$
*θ in **radians***



Arc and sector $r = 8$, $\theta = 1.2$: $l = 9.6$, $A = 38.4$, perimeter $8 + 8 + 9.6 = 25.6$.

Segment = sector − triangle $= 38.4 - \frac{1}{2}(64)\sin 1.2 \approx 38.4 - 29.83 = 8.57$.

Unknown angle a sector of radius 5 has area 15: $25\theta/2 = 15$, $\theta = 1.2$ rad.

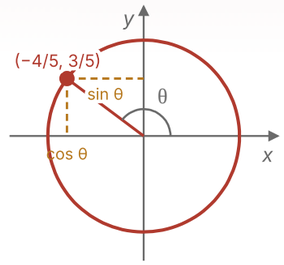
Watch out the perimeter of a sector includes **both radii** · calculator in radians for sin 1.2.

V Unit circle and identities

AA 3.5 · 3.6

$\cos\theta = x$, $\sin\theta = y$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$



θ	0	π/6	π/4	π/3	π/2
sin θ	0	1/2	√2/2	√3/2	1
cos θ	1	√3/2	√2/2	1/2	0

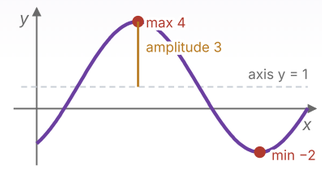
Obtuse θ, $\sin\theta = 3/5$: $\cos\theta = -4/5$ (second quadrant), $\sin 2\theta = -24/25$, $\cos 2\theta = 7/25$.

Watch out the sign comes from the **quadrant**, not from the square root · $\sin 2\theta \neq 2\sin\theta$.

VI Trigonometric graphs and equations

AA 3.7 · 3.8

$y = a\sin(b(x - c)) + d$
amplitude $|a|$ · period $2\pi/b$ · axis $y = d$ · max $d + |a|$



Read $y = 3\sin(2(x - \pi/6)) + 1$: amplitude 3, period π, axis y = 1, max 4, min −2.

Equation in 2x $2\cos 2x = 1$, $0 \le x \le \pi$: $0 \le 2x \le 2\pi$, $2x = \pi/3$ or $5\pi/3$, so $x = \pi/6, 5\pi/6$.

Quadratic in sin $2\sin^2 x - \sin x - 1 = 0$ on $[0, 2\pi]$: $(2\sin x + 1)(\sin x - 1) = 0$: $x = \pi/2$, $7\pi/6$, $11\pi/6$.

Watch out change the interval **with** the argument ($0 \le 2x \le 2\pi$) · $\sin x = 2$ has no solution: say so.

Unlocks $\cos 2x$ and $\sin x$ together → $\cos 2x = 1 - 2\sin^2 x$, a quadratic in sin x.

Command terms

Write down from the unit circle or the graph · **Show that** an identity, side to side · **Find the exact value** surds and π, no decimals · **Solve** every solution in the interval.

Paper 1 · Paper 2

P1, no GDC: exact values, identities, radians and arcs with π, equations with special angles.
P2, GDC: sine and cosine rules, 3D angles, ambiguous case, models y = a sin(b(x − c)) + d, intersections.

Booklet or memory?

booklet sine rule, cosine rule, ½ab sin C · l = rθ, A = ½r²θ · volumes and areas of solids · sin 2θ, cos 2θ · sin²θ + cos²θ = 1
by heart the exact values · the quadrant signs · degrees vs radians · the ambiguous case · amplitude, period, axis

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