Integral calculus

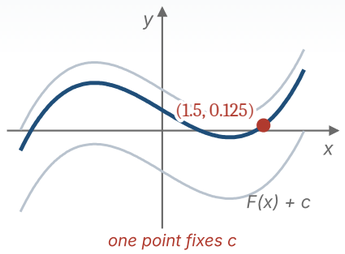
antidifferentiate → substitute → evaluate → area under → area between → motion

I Antiderivatives

AA 5.5 · 5.10

F is an antiderivative of f if $F' = f$. All of them differ by a **constant** :

$$\int f(x)\,dx = F(x) + c$$



one point fixes c

The standard integrals booklet 5.5 · 5.10

$f(x)$	$x^n,\ n \neq -1$	$\frac{1}{x}$	e^x	$\sin x$	$\cos x$
$\int dx$	$\frac{x^{n+1}}{n+1}$	$\ln\lvert x\rvert$	e^x	$-\cos x$	$\sin x$

Rewrite first: $\sqrt{x} = x^{1/2}$, $\frac{1}{x^2} = x^{-2}$, split a fraction term by term. **One point fixes** c.

Watch out no **+ c**, no mark · $\int fg \neq \int f \times \int g$ — expand first.

II Integration by substitution

AA 5.10

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du \quad \text{with } u = g(x)$$

the reverse of the chain rule

Four steps

1. choose u = the **inside** of the composite
2. $\frac{du}{dx}$ → replace dx; a missing **constant** goes outside
3. integrate in u
4. back to x — or, in a definite integral, **change the limits** and never come back

Recognise the pattern

$\int 2x(x^2+1)^5\,dx$	power of a bracket
$\int x\,e^{x^2}\,dx$	exponential, factor ½
$\int \frac{3x^2}{x^3+4}\,dx$	$\frac{g'}{g}$ → ln
$\int \sin^2 x \cos x\,dx$	trig: $u = \sin x$

Watch out $g'(x)$ must be there up to a **constant**: $\int e^{x^2}\,dx$ has no x outside — it cannot be done.

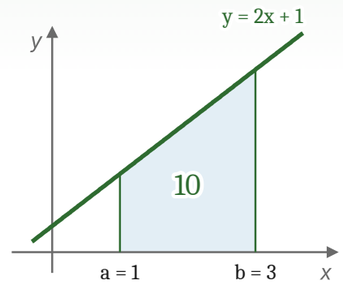
III The definite integral

AA 5.5 · 5.11

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

no + c: it cancels

$\int_1^3 (2x+1)\,dx = [x^2 + x]_1^3 = 12 - 2 = 10$ — the area of the trapezium.



Properties

$\int_a^a f = 0$ $\quad \int_b^a f = -\int_a^b f$ $\quad \int_a^b f + \int_b^c f = \int_a^c f$

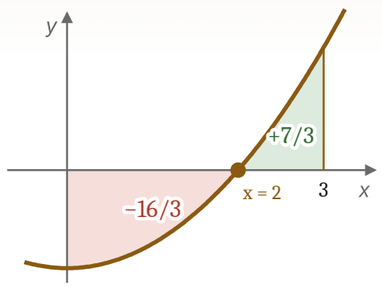
$\int_a^b (kf + g) = k\int_a^b f + \int_a^b g$ · $\int_a^b k\,dx = k(b-a)$, not k

Unknown limit or parameter: write the integral in terms of it, set it equal to the value, solve.

Paper 1 or Paper 2 GDC gives $\int_a^b$ directly on P2 · when the question says **exact** (or on P1): antiderivative, brackets, both limits shown — a decimal alone earns 0.

IV Area under a curve

AA 5.11



The integral counts area **above** the axis + and **below** it −.

$$\text{area} = \int_a^b |y|\,dx$$ booklet 5.11

$y = x^2 - 4$ on $[0, 3]$: integral $-16/3 + 7/3 = -3$, area $16/3 + 7/3 = 23/3$.

Method by hand

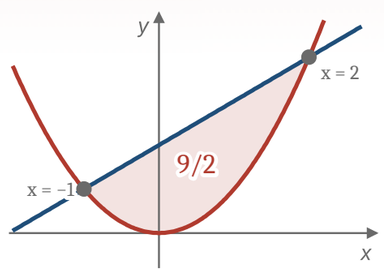
1. find the **roots** inside $[a, b]$
2. integrate **separately** on each piece
3. add the **sizes** (absolute values)

Watch out an integral of 0 does not mean an area of 0 · the region is bounded by the curve, the x-axis **and** the lines given — read all of them.

Unknown limit ($x = a$, $x = \ln k$): area as a function of it = value → solve.

V Area between two curves

AA 5.11



top: $y = x + 2$ — bottom: $y = x^2$

$$A = \int_a^b (\text{top} - \text{bottom})\,dx$$

a, b = the x-coordinates of the intersections

$x^2 = x + 2 \to x = -1, 2 \to \int_{-1}^{2} (x + 2 - x^2)\,dx = \frac{9}{2}$

1. **Intersections**: solve $f(x) = g(x)$ (by hand or GDC, store the full values)
2. **Which is on top?** test one x between them
3. the curves **cross** again → one integral per region, then add

Common shapes: line + parabola · two curves meeting twice · **three boundaries** (a curve, a line and an axis: split where the bottom changes) · a curve and its **tangent**.

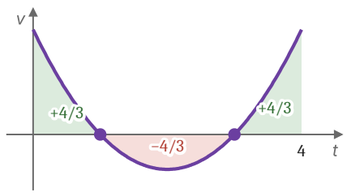
Watch out do not integrate bottom − top and then drop the minus sign: get the set-up right · on P2, use the stored intersections, not 0.627.

VI Kinematics

AA 5.9



$v = \frac{ds}{dt}$ $\quad a = \frac{dv}{dt}$ — integrating back needs a **condition** ($s(0)$, $v(0)$) to fix c



at rest at $t = 1$ and $t = 3$

$$\text{displacement} = \int_{t_1}^{t_2} v\,dt$$

$$\text{distance} = \int_{t_1}^{t_2} |v|\,dt$$ booklet 5.9

$v = t^2 - 4t + 3$, $0 \le t \le 4$: displacement $4/3$ m, distance **4 m**.

- **at rest**: $v = 0$ · **changes direction**: v changes **sign** · **speed** = $|v|$ · greatest speed: check the ends too

Watch out negative velocity ≠ slowing down: speeding up when v and a have the **same sign** · units: m, m s^{-1}, m s^{-2}.

Command terms

Find working carries the marks · **Find the exact value** antiderivative + limits, no decimal · **Show that** every line, the result is given · **Hence** use the part before · **Write down** the integral (set-up) is often 1 mark on its own.

Paper 1 · Paper 2

P1, no GDC: standard integrals, substitution, exact areas (ln, e, π, surds), kinematics with polynomials or trig.
P2, GDC: $\int$ and $\int|v|$ directly, intersections, unknown limit by solving. Store values, 3 s.f.

Booklet or memory?

in the booklet standard integrals · area $\int|y|$ · distance $\int|v|$
by heart + c · change the limits · top − bottom · split at the roots · $\int k\,dx = k(b-a)$

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