Lines, quadratics, exponentials and logs

line → parabola → discriminant → grow → invert → intersect

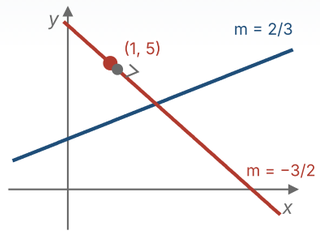
I Straight lines

AA 2.1 · 3.1

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $y - y_1 = m(x - x_1)$

$y = mx + c$ · $ax + by + d = 0$

parallel: $m_1 = m_2$ · perpendicular: $m_1 m_2 = -1$



distance $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$ · midpoint: the means of the coordinates

Perpendicular through a point $2x - 3y + 6 = 0$ has $m = \frac{2}{3}$; through (1, 5): $y - 5 = -\frac{3}{2}(x - 1)$, so $3x + 2y - 13 = 0$.

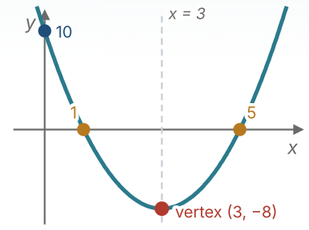
Intersection $y = 2x - 1$ and $3x + 2y = 12$: $3x + 4x - 2 = 12$, (2, 3) · **midpoint and distance** of (−1, 4), (5, −4): (2, 0), 10.

Watch out read m from $ax + by + d = 0$ by making y the subject: $m = -a/b$ · "in the form ax + by + d = 0" means **integers**.

II Quadratic functions

AA 2.6

$ax^2 + bx + c$	y-intercept c · axis $x = -b/2a$
$a(x - h)^2 + k$	vertex (h, k)
$a(x - p)(x - q)$	x-intercepts p, q



All three $f(x) = 2x^2 - 12x + 10 = 2(x - 3)^2 - 8 = 2(x - 1)(x - 5)$: vertex (3, −8), roots 1 and 5, $f(0) = 10$; range $f(x) \geq -8$.

Complete the square: factor a out of the x terms, halve the coefficient of x, square, subtract. **From a graph**: roots given → $a(x - p)(x - q)$, then one more point gives a.

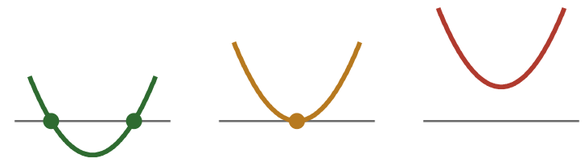
Watch out vertex of $2(x - 3)^2 - 8$ is (3, −8), not (−3, −8) · a < 0 opens downwards: the vertex is a **maximum**.

III Discriminant and inequalities

AA 2.7

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$, $\Delta = b^2 - 4ac$

"real roots": Δ ≥ 0 · "distinct": Δ > 0 · "equal": Δ = 0 · "no real roots": Δ < 0



Equal roots $x^2 + (k - 2)x + 4 = 0$: $(k - 2)^2 - 16 = 0$, $k = 6$ or $k = -2$.

Line tangent to a curve $y = mx + 1$ and $y = x^2 + 2$: $x^2 - mx + 1 = 0$, one root: $m^2 - 4 = 0$, $m = \pm 2$.

Inequality $x^2 - 2x - 15 < 0$: $(x - 5)(x + 3) < 0$, the parabola is **below** the axis between the roots: $-3 < x < 5$.

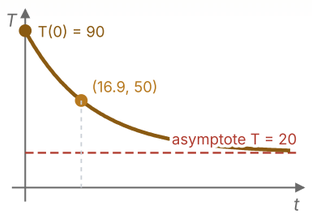
Watch out $kx^2 + 4x + k$: $\Delta = 16 - 4k^2$, and k ≠ 0 · sketch before answering an inequality.

IV Exponential functions and models

AA 2.9 · 1.5

$y = ka^x + c$ · $y = ke^{rx} + c$

asymptote $y = c$ · r > 0 growth, r < 0 decay · $a^x = e^{x \ln a}$



Cooling $T(t) = 20 + 70e^{-0.05t}$ °C: initially $T(0) = 90$; in the long run T tends to 20, the room temperature.

When is T = 50? $e^{-0.05t} = \frac{3}{7}$, $t = 20 \ln \frac{7}{3}$ ≈ 16.9 minutes.

Find the parameters: one value at t = 0 gives k + c, the long-run value gives c, a third value gives r (take ln).

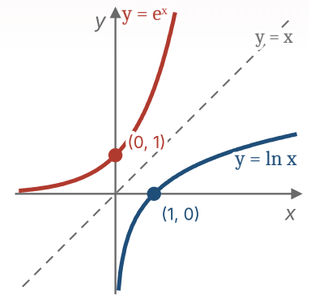
Watch out "initial" is t = 0 · the asymptote is reached in the long run, **never** exactly · $e^{-0.05t} = 3/7$ → divide first, then ln.

V Logarithmic functions

AA 2.9 · 2.5

$y = \log_a x$ is the inverse of $y = a^x$

reflection in $y = x$ · domain $x > 0$ · asymptote $x = 0$ · passes through (1, 0)



Features $y = \ln(x - 2) + 1$: domain $x > 2$, asymptote $x = 2$, x-intercept $\ln(x - 2) = -1$, $x = 2 + e^{-1} \approx 2.37$.

Inverse swap and solve: $x = \ln(y - 2) + 1$, so $f^{-1}(x) = e^{x-1} + 2$, range of f^{-1}: $y > 2$.

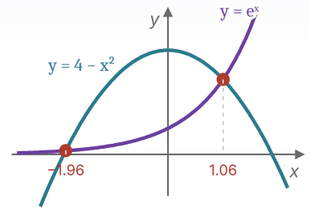
Watch out the domain of ln(…) comes from "inside > 0", never from the graph window · the domain of f is the range of f^{-1}.

VI Solving equations graphically

AA 2.10

1. $f(x) = g(x)$: graph $y = f(x)$ and $y = g(x)$
2. intersect, one point at a time
3. or graph $y = f(x) - g(x)$ and find its **zeros**
4. give every solution, to 3 s.f.

No algebra possible $e^x = 4 - x^2$: $x \approx -1.96$ or $x \approx 1.06$.



When is it graphical? x both in and out of a power, a log and a polynomial, "use your GDC", "to 3 s.f." — and on Paper 1 a **sketch** counts the solutions: two crossings, two solutions.

the question says	you do
"how many solutions"	count intersections on a sketch
"for what values of k"	slide a horizontal line y = k

Watch out zoom out: a second intersection may be off screen · set the window to the domain given.

Command terms

Write down read from the form or the graph · **Sketch** shape, intercepts, vertex, asymptote · **Show that** the given form at the end · **Find the range of values of k** an inequality on Δ · **Solve** every solution.

Paper 1 · Paper 2

P1, no GDC: completing the square, Δ with a parameter, exact ln answers, inverse of an exponential or a log.
P2, GDC: models with e, intersections, zeros, solutions to 3 s.f.

Booklet or memory?

booklet quadratic formula · $\Delta = b^2 - 4ac$ · gradient formula, $y - y_1 = m(x - x_1)$ · log laws
by heart the three forms of a quadratic · $m_1 m_2 = -1$ · domain of ln: inside > 0 · asymptote of $ke^{rx} + c$ is $y = c$

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