Number and algebra

write → add → multiply → invest → solve powers → expand

I Standard form and proof

AA 1.1 · 1.6

$a \times 10^k$, $1 \le a < 10$, k an integer

multiply: the a's together, add the powers — then put back into standard form

Calculate $(3.2 \times 10^5) \times (4 \times 10^{-2}) = 12.8 \times 10^3 = 1.28 \times 10^4$ · $(6 \times 10^8) \div (1.5 \times 10^{-3}) = 4 \times 10^{11}$.

Show that — a deductive proof

1. start from **one side** (usually the more complicated)
2. expand, simplify, one step per line
3. arrive at the **other side**, word it: "= RHS, as required"

Show that $(n+3)^2 - (n-3)^2 \equiv 12n$: LHS = $n^2 + 6n + 9 - (n^2 - 6n + 9) = 12n$ = RHS.

Prove the sum of two consecutive odd numbers is a multiple of 4: $(2n+1) + (2n+3) = 4n + 4 = 4(n+1)$.

Watch out 12.8×10^3 and 0.45×10^6 are not in standard form · never work on both sides at once, and never start from what you must prove.

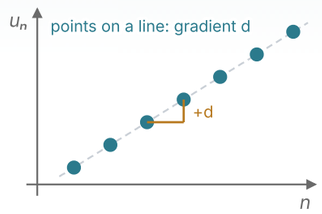
II Arithmetic sequences and series

AA 1.2

$u_n = u_1 + (n-1)\mathrm{d}$

common difference $d = u_{n+1} - u_n$

$S_n = \frac{n}{2}(2u_1 + (n-1)\mathrm{d}) = \frac{n}{2}(u_1 + u_n)$



Two terms given $u_3 = 11$, $u_8 = 31$: $5\mathrm{d} = 20$, $\mathrm{d} = 4$, $u_1 = 3$, so $u_n = 4n - 1$.

Sum $S_{20} = 10(6 + 76) = 820$ · **which term** is 199? $4n - 1 = 199$, $n = 50$ · $\Sigma_{k=1}^{10}(2k+1) = 120$ (10 terms, first 3, last 21).

In context: rows of seats, a salary rising by a fixed amount, a stack — "the same amount each time".

Watch out n must be a positive **integer**: $S_n = n(2n+1) > 500$ gives $n > 15.6$ → the answer is **16** · S_n is a sum, u_n a single term.

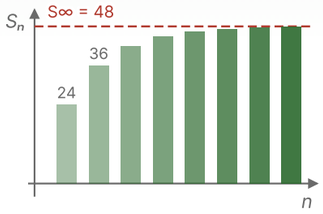
III Geometric sequences and series

AA 1.3 · 1.8

$u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(1 - r^n)}{1 - r}$

$S_\infty = \frac{u_1}{1 - r}$

only if $|r| < 1$



Two terms given $u_2 = 12$, $u_5 = 1.5$: **divide**, $r^3 = \frac{1}{8}$, $r = \frac{1}{2}$, $u_1 = 24$, $S_\infty = 48$.

Sum $S_6 = 48(1 - (0.5)^6) = 47.25$ · **least n** with $S_n > 47.9$: $0.5^n < 0.1/48$, $n > 8.91$, so **n = 9** (check: $S_8 = 47.81$, $S_9 = 47.91$).

Watch out divide, never subtract, to find r · a negative r alternates the signs · no S_∞ when $|r| \ge 1$: say why.

Unlocks unknown power n → logarithms or a table on the GDC; "converges" → write $|r| < 1$ and solve for the parameter.

IV Financial applications

AA 1.4

$\mathrm{FV} = \mathrm{PV} \times (1 + \frac{r}{100k})^{kn}$

k compounding periods per year, n years · depreciation: $V = V_0(1 - r/100)^n$

Compound interest \$5000 at 3.2 % a year, compounded monthly, 6 years: $5000(1 + 0.032/12)^{72} = 6056.80$.

Depreciation a car bought at \$24 000 loses 15 % a year: after 4 years $24000 \times 0.85^4 = 12528.15$.

How long to double at 4 % compounded quarterly? $1.01^{4n} = 2$, $n = 17.4$ → **18** complete years.

Real value with 2 % inflation a year: $6056.80 \div 1.02^6 \approx 5378$ in today's money.

TVM solver	N = total periods (kn) · I% = annual rate · PV **negative** (money paid in) · PMT = 0 · P/Y = C/Y = k

Watch out PV and FV have opposite signs in the solver · round money to 2 d.p. only at the end · "complete years" rounds **up**.

V Exponents and logarithms

AA 1.5 · 1.7

$a^m a^n = a^{m+n}$ · $(a^m)^n = a^{mn}$ · $a^{-n} = 1/a^n$ · $a^{1/n} = \sqrt[n]{a}$

$\log_a x = y \Leftrightarrow a^y = x$

$\log(xy) = \log x + \log y$ · $\log(x/y) = \log x - \log y$ · $\log x^m = m \log x$ · $\log_a x = \ln x / \ln a$

Same base $8^x \times 4^{1-x} = 2^{3x} \times 2^{2-2x} = 2^{x+2}$.

Unknown power $3^{2x-1} = 20$: $2x - 1 = \frac{\ln 20}{\ln 3}$, $x = 1.86$ (3 s.f.).

Log equation $\log_2 x + \log_2(x-2) = 3$: $x(x-2) = 2^3$, $x = 4$ or $x = -2$: **reject −2**, log of a negative number.

Watch out $\log(x+y) \ne \log x + \log y$ · $(2x^3)^4 = 16x^{12}$, not $2x^{12}$ · check every root in the original equation.

Unlocks unknown in a power → take ln of both sides; several logs → combine into one, then remove the log.

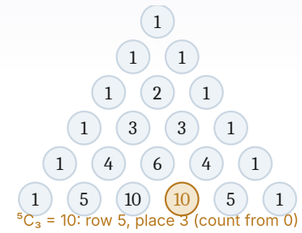
VI The binomial theorem

AA 1.9

$(a+b)^n = \Sigma_{r=0}^{n}\, {}^nC_r\, a^{n-r}b^r$

${}^nC_r = \frac{n!}{r!(n-r)!}$ · *the general term*

1. write the **general term** with the brackets
2. collect the powers of x
3. choose r from the power wanted



$^5C_3$ = 10: row 5, place 3 (count from 0)

A coefficient x^3 in $(2x-3)^5$: ${}^5C_3(2x)^3(-3)^2 = 10 \times 8 \times 9 = 720$ — so 720.

Term independent of x in $(x^2 + 2/x)^6$: general term ${}^6C_r x^{12-2r} 2^r x^{-r}$, $12 - 3r = 0$, $r = 4$: $15 \times 16 = 240$.

Watch out the bracket takes the power: $(2x)^3 = 8x^3$ · the sign of $(-3)^r$ · the coefficient is a number, without the x.

Command terms

Write down straight from the formula · **Show that** LHS to RHS, every line · **Prove** a general argument, with n · **Find the least value of n** an integer, with a check · **Hence** the previous answer is the key.

Paper 1 · Paper 2

P1, no GDC: standard form, exact logs, laws of exponents, nCr from Pascal, sequences with integer answers.
P2, GDC: TVM solver, compound interest and depreciation, least n by table, ln with decimals.

Booklet or memory?

booklet u_n and S_n of both sequences · S_∞ · compound interest · log laws and change of base · binomial theorem, nCr
by heart how to **use** them: divide two geometric terms, $|r| < 1$, integer n, TVM signs, reject the impossible root

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